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Dynamic ridesharing with variable-ratio charging-compensation scheme for morning commute

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ABSTRACT

This paper investigates the dynamic ridesharing with the variable-ratio charging-compensation scheme (VCS) in morning commute, with the continuous-time point-queue model applied to a single bottleneck. The optimal VCS without imposing road pricing when the ridesharing platform minimizes the disutility or maximizes its profit is analyzed. It is found that the user equilibrium coincides with the system optimum when the platform minimizes the system disutility with VCS, and the corresponding platform's profit is negative with high travel demand. Considering this, the optimal VCS when the platform minimizes the system disutility with zero profit is examined. Moreover, to ensure ridesharing participants commute with no queue, they need to depart at the two tails of the departure time window. Under that case, the optimal VCS are investigated with desirable objectives of the ridesharing platform. The analytical results indicate there should be fewer commuters involved in ridesharing when the platform maximizes its profit compared to that when the platform minimizes the system disutility with zero profit.

1. Introduction

1.1. Background

Traffic congestion, usually accompanied with rapid urbanization and growth in private auto ownership, is becoming an increasingly critical problem worldwide. The induced lost times, disrupted schedules, wasted fuel, polluted environment, and other discomforts can lead to significantly reduced quality of life. For example, it was reported that traffic congestion has cost Americans in urban areas approximately $305 billion in 2017 (Rahim, 2018). Among the methods of travel demand management, ridesharing is promoted as a way to better utilize the empty seats in most passenger cars, thus lowering fuel usage and transport costs. Ridesharing is a service that arranges shared rides between drivers and passengers with similar itineraries in which ridesharing passengers make compensations (i.e., payments) to drivers for their trips. Ridesharing has been practiced for decades and was also referred to as carpooling in the past. Data showed that there used to be 20.4% of American workers commuted to work by carpool, and that percentage has declined to 10.7% in 2008 (Chan and Shaheen, 2012). Currently, dynamic ridesharing (also known as real-time ridesharing) services have emerged due to the recent advances of mobile Internet services combined with the GPS technology. GPS can help locate travelers and drivers

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quickly and accurately, while mobile Internet services provide mobile platforms in which passengers can request rides and drivers can respond to the requests. The platform also makes it easier and more efficient to handle payments and match rides. Section 1.2 below provides a brief review on the ridesharing literature. For more comprehensive reviews and future research directions of ridesharing, one may also refer to Agatz et al. (2012), Furuhata et al. (2013) and Mourad et al. (2019).

How passengers pay to receive ridesharing services and how drivers are compensated is an important component of ridesharing. Among the various ridesharing charging and compensation schemes, the fixed-ratio charging-compensation scheme (FCS) is perhaps the most common scheme studied so far. Generally, FCS refers to the scheme in which a fixed percentage (e.g., 80% or 100%) of what the ridesharing passengers pay is transferred to the driver who carries them as the compensation. This setting simplifies the analysis of ridesharing, which however does not offer much flexibility for the ridesharing platform to achieve its desirable objectives, such as to minimize the system's costs or to maximize the platform's profit. Indeed, certain road pricing schemes or parking charges have been usually assumed in the literature on top of ridesharing in order to achieve such objectives (e.g., Liu and Li, 2017; Ma and Zhang, 2017).

In this paper, we aim to answer the following questions: can we design more flexible charging and compensation schemes for ridesharing, which can help achieve desirable objectives of the ridesharing platform without imposing additional road pricing schemes? We show that the answer to the questions is affirmative by the so-called variable-ratio charging-compensation scheme (VCS). VCS means the fees paid by passengers for their trips do not need to be directly related to the payments received by the drivers. In this way, FCS is just a special case of VCS. Compared to the FCS, VCS allows more flexibility for the platform to achieve its desirable system objectives. To the authors' best knowledge, most existing ridesharing researches assume FCS, and so far VCS has not been proposed/studied for ridesharing. In this paper, we introduce VCS into and study its impact on ridesharing.

We assume a ridesharing platform behaves as an independent agent who applies the VCS to optimize certain objectives: minimizing the system disutility, maximizing the platform's profit, and minimizing the system disutility with zero profit of the platform. We investigate when and how the system disutility and platform's profit can be impacted by ridesharing and the VCS. Here system disutility is defined as the difference between the total travel costs and the platform's profit. The value of total travel costs is the sum of the commuting costs of all travelers; the platform's profit is computed by the integral of the difference between the charges from the passengers and the compensation to the drivers for the entire study period. In particular, we study the dynamic ridesharing system for morning commute on a single bottleneck (Vickrey, 1969). We assume fixed travel demand and desired arrival time, and that identical travelers commute to the destination each morning by cars. The travel time cost and schedule delay cost are both considered in the commuter's departure time choice, and no one can reduce his/her costs by changing the departure time at the equilibrium. In the traditional morning commute problem with bottleneck models, the system optimum with minimum travel cost can be obtained by time-varying tolls to eliminate the queuing congestion (e.g., Arnott et al., 1990a,b; Arnott and Kraus, 1998). However, there are strong public/political resistances to the implementation of road pricing (Evans, 1992; Arnott et al., 1994; Hau, 1998; Taylor and Kalauskas, 2010). In this paper, we will study whether the system optimum can be reached with VCS without road pricing.

We summarize the major contributions of this paper as follows:

(1) We introduce and integrate VCS to dynamic ridesharing in the morning commute problem with a single bottleneck. We investigate the optimal VCS under different objectives of the platform: minimizing the system disutility, maximizing the platform's profit, and minimizing the system disutility with platform's zero profit.
(2) We analyze the optimal VCS when ridesharing participants commute with no queue, which can be solved analytically.
(3) We obtain interesting findings through the analyses, e.g., (i) the fuel cost and the inconvenience cost parameters for ridesharing drivers and passengers play a vital role in the analyses; (ii) to ensure ridesharing participants commute with no queue, they need to depart at the two tails of the departure time window; (iii) VCS can enable a ridesharing platform to achieve its (various) desirable objectives, without the use of road pricing. These findings are expected to provide useful insights for the ridesharing platform to operate and manage dynamic ridesharing systems by designing certain VCS.

The remainder of the paper is organized as follows. Section 1.2 provides a review of the existing ridesharing literature. Section 2 gives the main introduction of the morning commute problem when the ridesharing platform adopts VCS. In Section 3, the optimal VCS under three objectives of the ridesharing platform, minimizing the system disutility, maximizing the profit, and minimizing the system disutility with zero profit, are investigated. Section 4 discusses a special case when the ridesharing participants commute with no queue. Section 5 conducts numerical examples and Section 6 concludes the paper with discussion of future research.

1.2. Literature review

Studies on ridesharing can be broadly categorized into two major groups: one investigates the driver-passenger matching problem from the perspective of the entire system, which is often formulated as an optimization problem, while the other examines travelers' decision making regarding ridesharing and the induced impact on traffic congestion (Liu and Li, 2017).

As reviewed by Agatz et al. (2012), with ridesharing introduced into the transportation system, the matching problem has mainly focused on the following aspects: to minimize the system-wide vehicle-miles, to minimize the system-wide travel time, or to maximize the number of participants. For example, to reduce the total travel cost and to maximize the matched carpooling participants, Baldacci et al. (2004) proposed an integer programming formulation, and developed both exact and heuristic solution methods. Calvo et al. (2004) constructed an integrated information system for the carpooling service, and examined the impacts of varying the ratio of drivers to passengers on the total travel time using real-life carpooling data. Agatz et al. (2011) studied how to minimize the total travel distance and total travel time with ridesharing. Their simulation studies showed that dynamic ridesharing may have the potential for success in large US metropolitan areas. Guasch et al. (2014) examined the goal of minimizing the total waiting time. To improve the matched trips and participants, Stiglic et al. (2015) optimized the meeting points in ridesharing systems. Those existing studies, focusing more on optimizing certain objectives of the ridesharing system, have made significant contributions to the understanding of the transportation system with ridesharing. Recently, Long et al. (2018) proposed a stochastic ridesharing model to examine the travel time uncertainty. They developed a bi-objective ridesharing matching model and explored the importance of considering travel time uncertainty when determining the matches. To the best of our knowledge, however, most of the studies are limited to static cases and lack further analyses on traveler's decision making. The handful studies on dynamic ridesharing are mainly restricted to simulation analysis, which cannot provide deeper analyses of the traffic flow pattern, system-wide travel time, and the number of participants due to ridesharing.

When traveler's decision making is considered, Yang and Huang (1999) and Huang et al. (2000) are among the first to establish the mode choice equilibrium when carpooling is introduced with high-occupancy vehicles (HOV)/high-occupancy toll (HOT) and general-purpose (GP) lanes. To examine the impacts on the carpooling behavior, fuel consumption, sharing cost, value of time, and individual preferential or attitudinal attributes were considered in Huang et al. (2000). They found that an externality-based congestion toll is necessary when carpooling is sensitive to traffic congestion reduction. Qian and Zhang (2011) studied the morning commute problem with carpool when HOV lanes were introduced into the one-to-one corridor network. They analyzed how different factors may affect travelers' mode choice and the resulting network performance. Xiao et al. (2016) investigated the carpooling behavior under parking space constraints at trip destinations in the morning commute problem. They found that carpoolers have shorter departure time window compared with solo drivers in order to smooth the extra carpool cost. Notice that the above studies simply assumed the extra carpool cost as a composite parameter and the charging-compensation scheme was not introduced due to the unavailability of mobile Internet services.

The recent research on ridesharing has paid more attention to the role and impact of the charging-compensation scheme. With the introduction of FCS, Xu et al. (2014) analyzed how traffic congestion can be impacted by ridesharing charging scheme and how people can be motivated to involve in ridesharing. Xu et al. (2015) further developed a static traffic equilibrium model with FCS on a general network by considering ridesharing as a separate mode, and formulated the user equilibrium as a mixed complementarity problem. Liu and Li (2017) examined the time-varying compensation scheme design in a single bottleneck corridor for ridesharing user equilibrium. Further, they explored the time-varying tolls to help reach the system optimum. Ma and Zhang (2017) studied the traffic flow patterns in a single bottleneck corridor under various scenarios with different ridesharing charging and shared parking prices. They relaxed the assumption adopted in Liu and Li (2017) with constant ridership for each ridesharing vehicle (i.e., a ridesharing driver can only pick up one passenger to form a two-person ridesharing trip), and found that the departure rates and travel time functions may be nonlinear with a dynamic ridesharing mode and dynamic parking charges, which were not observed in Arnott et al. (1990b). To achieve better system performance, the dynamic parking charges combined with constant ridesharing payments can be selected to eliminate queuing delays. On the basis of Xu et al. (2015), Di et al. (2018) developed a mathematical program to explore the network design with FCS. In particular, they extended the ridesharing user equilibrium to accommodate HOT lane deployment and tolling.

Note that all the previous studies reviewed here adopted the FCS. The models were either conducted on general networks or corridor networks, and mainly focused on traffic assignment or charging scheme design. Table 1 provides a systematic comparison of the key modeling components in the existing ridesharing research with consideration of the traveler's decision making. It is clear from the table that most existing studies examined the traffic flow pattern when ridesharing is introduced into the network. However, under FCS, the performance of system disutility and the platform's profit cannot be further examined by these existing studies. Furthermore, majority of the studies aimed to solve the traffic assignment

Table 1
Related studies on ridesharing with consideration of travelers' decision making.

Paper	Model setting	Problem type	Ridership for each ridesharing vehicle	Road pricing
Yang and Huang (1999)	Corridor network	Static traffic assignment	Fixed	Yes
Huang et al. (2000)	Corridor network	Static traffic assignment	Fixed	Yes
Qian and Zhang (2011)	Corridor network	Dynamic traffic assignment	Fixed	Yes
Xu et al. (2014)	General network	Static traffic assignment	Variable	No
Xu et al. (2015)	General network	Static traffic assignment	Variable	No
Xiao et al. (2016)	Corridor network	Dynamic traffic assignment	Fixed	Yes
Liu and Li (2017)	Corridor network	Dynamic traffic assignment	Fixed	Yes
Ma and Zhang (2017)	Corridor network	Dynamic charging scheme design	Variable	Yes
Di et al. (2018)	General network	Static traffic assignment	Variable	Yes

problem and to find the appropriate road pricing scheme for achieving the social optimum. Road pricing however may not be easily implemented in practice due to possible public/political opposition. In this paper, we introduce VCS to dynamic ridesharing and investigate the optimal VCS in order to achieve different system objectives of the ridesharing platform.

2. Model framework

In this section, we present our modeling framework for the morning commute problem (with a single bottleneck) with a ridesharing platform. Behaviors of three agents, namely the solo drivers, ridesharing drivers, and ridesharing passengers, are considered. VCS is adopted by the ridesharing platform, in which the platform may charge the ridesharing passengers differently from what it compensates the ridesharing drivers. For different objectives of the platform, the user equilibrium states when all travelers are subject to identical commuting costs are investigated.

2.1. Notations

The notations used in this paper are listed as follows.

Model parameters (all positive scalars)

α	value of time
β	unit cost of early arrival penalty
γ	unit cost of late arrival penalty
t^*	desired arrival time
s	service rate or capacity of the bottleneck, veh/h
f	fuel cost parameter
h_r	inconvenience cost parameter for ridesharing drivers
h_p	inconvenience cost parameter for ridesharing passengers
N	total commuting demand

Time-varying variables

$q(t)$	queue length at the bottleneck at time t
$u(t)$	departure rate of the vehicular traffic (including solo drivers and ridesharing drivers) at time t
$u_s(t)$	departure rate of the vehicular traffic for solo drivers at time t
$u_r(t)$	departure rate of the vehicular traffic for ridesharing drivers at time t
$\tau(t)$	travel time for travelers departing at time t
$c_s(t)$	generalized travel cost of solo drivers who depart from origin at time t
$c_r(t)$	generalized travel cost of ridesharing drivers who depart from origin at time t
$c_p(t)$	generalized travel cost of ridesharing passengers who depart from origin at time t
$c(t)$	minimum travel cost of drivers (including solo drivers and ridesharing drivers) who depart from origin at time t
$m_p(t)$	the fare charged by the platform for each passenger departing from origin at time t
$m_r(t)$	the fare compensated by the platform for each driver departing from origin at time t

Intermediate notations

t_1	the earliest departure time from origin
t_2	the critical departure time from origin, $t_2+\tau(t_2)=t^*$
t_3	the latest departure time from origin
t_4	the earliest departure time from origin for solo drivers
t_5	the latest departure time from origin for solo drivers
N_s	travel demand for solo drivers, veh
N_r	travel demand for ridesharing drivers, veh
T_s	the departure time window for solo drivers
T_r	the departure time window for ridesharing drivers
T	the whole study period, with relation $T=T_s \cup T_r=[t_1,t_3]$ holds
$\widehat{c}$	the identical generalized travel cost at user equilibrium
π	the profit of the platform
G	the system disutility, defined as the difference between total travel costs and profit of the platform

2.2. Main assumptions

Before introducing the model framework, the main assumptions in this study are listed here.

(A1). The total travel demand is fixed and given. Commuters are assumed to be homogenous, from the value of time to the schedule delay penalties. All travelers have their own cars, and are subject to identical desired arrival time at the (single) destination.

(A2). The ridesharing platform adopts the VCS for commuters on shared trips. In accordance with the assumption of piecewise linear travel time and queuing delays, and the piecewise constant departure rates in the traditional bottleneck model for the morning commute problems, charging and compensation are assumed as piecewise linear functions with departure time.

(A3). A ridesharing driver can only pick up one passenger during the commute, i.e., the maximum ridership is fixed as 1 for each ridesharing vehicle. The ridesharing picking up and dropping off time is set as zero for both drivers and passengers to simplify the discussions.

(A4). Ridesharing drivers experience inconvenience for picking up passengers, and passengers also experience inconvenience for the shared ride. The perceived inconvenience cost is defined as a function of travel time, with a different inconvenience cost parameter for each type of agents.

(A5). All drivers need to pay for the fuel during their trips, with the same fuel cost parameter for all vehicles (drivers).

Notice that the assumption of fixed travel demand (A1) is also adopted in many previous studies (e.g., Xu et al., 2015; Liu and Li, 2017; Ma and Zhang, 2017; Di et al., 2018). Commuters have their own cars, and are free to be a solo driver, or a ridesharing driver or a passenger. The demand for each mode is determined by the equilibrium. Different from Xu et al. (2014, 2015), Liu and Li (2017), Ma and Zhang (2017) and Di et al. (2018), where they adopted the FCS, we assume in (A2) the ridesharing platform adopts the VCS to impose charges and compensations for drivers and passengers. The fixed ridesharing ridership for each vehicle, i.e., (A3), was also imposed by Liu and Li (2017). It is adopted here to simplify our analyses and generate linear departure rate functions with certain linear ridesharing charging-compensation schemes, which is in line with the traditional bottleneck model for the morning commute problem. In (A4), the inconvenience cost function defined in this paper is different from Xu et al. (2015) and Di et al. (2018), where they assumed the function is related to the ridesharing driver flow and ridesharing rider flow on each link traversed. For simplicity, we model the inconvenience cost as a function of the travel time. Together with the fuel loss experienced by the drivers in (A5), we can investigate the relation between the inconvenience cost and fuel cost.

2.3. Bottleneck model

The bottleneck model follows Vickrey (1969), where a single origin-destination (OD) pair is connected by a single link. There exists a bottleneck at the end of the link with a fixed capacity s. The link free-flow travel time is τ_0. The continuous-time point-queue model (e.g., Ban et al., 2012a) is adopted here to depict the queue dynamic as below

$$\dot{q}(t) = u(t - \tau_0) - w(t), \tag{1}$$

where $\dot{q}(t)$ is the change rate of queue length, $u(t)$ is the departure rate of vehicular traffic from origin at time t, and $w(t)$ is the exit flow rate of the bottleneck satisfying

$$w(t) = \begin{cases} u(t-\tau_0), & \text{if } q(t) = 0 \text{ and } u(t-\tau_0) < s \\ s, & \text{otherwise} \end{cases} \tag{2}$$

Eq. (2) indicates when there is no queue and simultaneously the flow rate is less than the capacity, the exit flow rate equals to the inflow rate with a free-flow travel time delay; otherwise the exit flow rate should be the bottleneck flow capacity.

Respecting the first-in-first-out rule, the total travel time for each vehicle departing from origin to destination at time t is

$$\tau(t) = \tau_0 + q(t+\tau_0)/s. \tag{3a}$$

With (3a), we get

$$\dot{\tau}(t) = \dot{q}(t+\tau_0)/s. \tag{3b}$$

Based on the traditional bottleneck model for the morning commute problems (Arnott et al., 1990a,b), both the queuing delays and travel time are (piecewise) linear functions with time t, while the departure rates are piecewise constant for early and late arrivals.

2.4. Variable-ratio charging-compensation scheme (VCS)

In our model, the ridesharing platform deals with three types of agents: solo drivers, ridesharing drivers, and ridesharing passengers. During the morning peak period, the platform charges each ridesharing passenger departing from the origin at time t a (time-varying) fee denoted as $m_p(t)$, and compensates each ridesharing driver with $m_r(t)$. In FCS, $m_r(t)=\rho m_p(t)$, where ρ is a fixed parameter, indicating that certain percentage of the payment from passengers is transferred to drivers as the compensation. In this way, there is only one variable (charging or compensation) to be determined by the model setting. In VCS, however, this ratio of the charging and compensation is not fixed. In Assumption (A2), linear charging and compensation functions with departure time t are assumed in this paper; see also Sections 3 and 4. Different from existing studies, the time-varying charging and compensation functions here are all endogenous (i.e., determined by the model), with their specific forms dependent on the objectives of the platform, e.g., (1) to minimize the system disutility; (2) to maximize the profit of the platform, and (3) to minimize the system disutility with zero profit of the platform. For each objective, the design of VCS is different, albeit they follow the general linear forms of time t. In this way, the charging and compensation in the proposed VCS model are all decision variables to be determined, which is fundamentally different from that of FCS in existing studies (Xu et al., 2015; Liu and Li, 2017; Ma and Zhang, 2017; Di et al., 2018). The endogenous VCS scheme can help the platform achieve their desirable objectives (in terms of system disutility and revenue), and thus may be more preferable by the platform.

Table 2
Cost function formulation for each type of agent.

	Travel time cost	Schedule delay penalties	Fuel cost	Charge	Compensation	Inconvenience cost
Solo drivers	√	√	√	–	–	–
Ridesharing drivers	√	√	√	–	√	√
Ridesharing passengers	√	√	–	√	–	√

2.5. Cost functions

Under the assumptions in Section 2.2, the travel cost function for solo drivers, ridesharing drivers and ridesharing passengers are formulated in this subsection.

For a solo driver, his/her travel cost is the summation of the travel time cost, schedule delay penalty, and fuel cost:

$$c_s(t) = \alpha\tau(t) + \max\{\beta(t^* - t - \tau(t)), \gamma(t + \tau(t) - t^*)\} + f\tau(t), \tag{4}$$

where α is the value of time, β and γ is the unit schedule delay penalty separately for early arrival and late arrival. Generally, it should hold that $\gamma > \alpha > \beta$ (Lindsey, 2004). t^* is the desired arrival time for all commuters, and f is the fuel loss parameter for all vehicles.

For a ridesharing driver, the inconvenience cost is accompanied with the compensation paid from the platform when they pick up the ridesharing passenger to the destination. Together with the travel time cost, schedule delay penalty, and fuel cost, the cost function is as follows:

$$c_r(t) = \alpha\tau(t) + \max\{\beta(t^* - t - \tau(t)), \gamma(t + \tau(t) - t^*)\} + f\tau(t) + h_r\tau(t) - m_r(t), \tag{5}$$

where $m_r(t)$ is the compensation from the platform for picking up the passenger, and h_r is the inconvenience cost parameter for ridesharing drivers.

Compared with ridesharing drivers, a ridesharing passenger should be charged by the platform for his/her ride. The cost function is depicted as follows:

$$c_p(t) = \alpha\tau(t) + \max\{\beta(t^* - t - \tau(t)), \gamma(t + \tau(t) - t^*)\} + h_p\tau(t) + m_p(t), \tag{6}$$

where $m_p(t)$ is the charge by the platform and h_p is the inconvenience cost parameter for ridesharing passengers. Specially, the inconvenience cost experienced by the ridesharing drivers is different from that of ridesharing passengers. Note the ridesharing passengers do not drive, and thus the fuel cost is not included in their cost function.

For each type of agents, the components of the cost function can be summarized in Table 2.

In summary, we will study the morning commute problem on a single bottleneck with VCS. For fixed travel demand, a single OD pair is connected by a solo link with a bottleneck at the end of the link, which means all travelers depart from the same origin and commute to the same destination for their commute. During the morning peak period, the ridesharing platform charges each passenger with a time-varying fee for his/her shared ride, and makes the time-varying compensations to a ridesharing driver for his/her pickup behavior. Since the VCS is examined in a single OD pair, and the pickup and pickoff times are not considered in this paper, then the assumption that the charging and compensation are linear functions with the departure time is reasonable. The values of charges may be not identical to the compensations, and both the ridesharing charges and compensations should be determined by the ridesharing platform to achieve its desirable objectives. The ridership in each ridesharing vehicle is assumed as 1 for simplicity. The aggregated impacts on traffic congestion only result from the number of drivers including solo drivers and ridesharing drivers. For solo drivers commuting alone to the destination, they make trade-offs between the cost of travel time, the cost of schedule delays (when they cannot arrive at the destination punctually) and the cost of fuel. For ridesharing drivers, the extra considerations are the inconvenience of riding with someone else and the compensations they receive. For ridesharing passengers, the generalized cost is composed of the cost of travel time, the cost of schedule delays, the fees charged by the ridesharing platform, and the inconvenience cost of sharing the rides. At equilibrium, nobody can reduce his/her own travel cost by unilaterally changing the departure time or mode choices.

In this paper, rather than studying the dynamic matching problem between ridesharing drivers and passengers, we examine the optimal time-varying charging-compensation schemes in dynamic ridesharing to achieve various objectives of the platform such as minimizing the total system disutility and maximizing the profit, without imposing road pricing (or parking charging schemes). The optimal VCS when ridesharing participants commute with no queue is also investigated in this paper.

2.6. General formulation of dynamic user equilibrium (DUE) with ridesharing on a single bottleneck

We first propose a general formulation for dynamic user equilibrium with ridesharing on a single bottleneck for the morning commute problem. At user equilibrium, the travel cost for commuters departing from the origin at time t ($t \in T$, where T is the whole study period) should be equal to each other, regardless of the departure time and the modes they choose. Under a given charging-compensation scheme, the continuous-time dynamic user equilibrium (DUE) with ridesharing can be formulated as a differential complementarity systems (DCS; see Pang and Stewart (2008), Ban et al. (2012b), and Ma et al. (2018)),

$$0 \le u_s(t) \perp c_s(t) - c(t) \ge 0, t \in T, \tag{7a}$$

$$0 \le u_r(t) \perp c_r(t) - c(t) \ge 0, t \in T, \tag{7b}$$

$$0 \le u(t) \perp c(t) - \widehat{c} \ge 0, t \in T, \tag{7c}$$

$$u(t) = u_s(t) + u_r(t), t \in T, \tag{7d}$$

$$u_r(t) = u_p(t), t \in T, \tag{7e}$$

$$c_r(t) = c_p(t), t \in T, \tag{7f}$$

where $u_s(t)$ and $u_r(t)$ are respectively the departure rates of vehicular traffic for solo drivers and ridesharing drivers at time t, which leads to $u(t) = u_s(t) + u_r(t)$ in Eq. (7d) (since ridesharing passengers do not drive the car). $u_p(t)$ and $c_p(t)$ are respectively the departure rates for ridesharing passengers and the corresponding travel cost. We have Eqs. (7e) and (7f) due to the assumption of fixed ridership (as 1) for each ridesharing vehicle. $c_s(t)$ and $c_r(t)$ are the corresponding travel cost functions defined in Section 2.5. $c(t)$ is the minimum travel cost for drivers (solo drivers or ridesharing drivers) departing at time t, and $\widehat{c}$ is the identical generalized travel cost for all travelers at DUE. Therefore, we have $c_s(t) \ge c(t)$, $c_r(t) \ge c(t)$ and $c(t) \ge \widehat{c}$. The symbol "⊥" in Eq. (7) stands for "perpendicular" such that $x \perp y \Leftrightarrow x^{\mathrm{T}} y = 0$. Eq. (7) states that if there are drivers (solo drivers or ridesharing drivers) departing at time t, then the travel cost must be minimal; for any departure time t not chosen by drivers (solo drivers or ridesharing drivers), the travel costs at that time must be larger than or equal to $\widehat{c}$. When the equilibrium is achieved, commuters choose the departure time and travel mode with minimal trip costs. Therefore, the complementarity systems in Eq. (7) formulate the DUE conditions.

In addition, since we assume the maximum ridership is 1 for each ridesharing vehicle, then if N_s and N_r denote the demands for solo drivers and ridesharing drivers, respectively, there must be $N_s + 2N_r = N$, which can be equivalently written as

$$\int_0^T u_s(t)\mathrm{d}t + 2\int_0^T u_r(t)\mathrm{d}t = N. \tag{8}$$

Eq. (8) does not depend on whether there are commuters involved in ridesharing or not. If there are ridesharing participants, there should be $N_s < N$, $N_r > 0$; otherwise, $N_s = N$, $N_r = 0$.

With the flow dynamics constraints in Eqs. (1)–(3), the travel cost functions in Eqs. (4)–(6), the complementarity systems in Eq. (7), and the conservation relation in Eq. (8), we have established the DCS formulation for continuous-time DUE on a single bottleneck under the given charging-compensation scheme. The solution existence can be investigated by following similar methods in Ban et al. (2012b) based on the general DCS theories in Pang and Stewart (2008). In this paper, we omit the relevant discussions in order to save space; instead we focus on exploring the special structure of the DUE formulation (since it is on a single bottleneck) to derive the model and solution properties.

2.7. A general bi-level formulation of VCS design

Compared to the ridesharing with FCS studied by Xu et al. (2015), Liu and Li (2017), Ma and Zhang (2017) and Di et al. (2018), it should be particularly noted that the DCS-based DUE formulated in this paper is developed under the given charging-compensation scheme, which should be designed under the platform's desirable objective such as to minimize the system disutility, maximize the profit of the platform, or minimize the system disutility with zero profit of the platform. Therefore, to find the optimal VCS, we need to solve the following optimization problem

$$\min_{m_r(t), m_p(t)} (\max) J, \tag{9}$$

subject to Eqs. (1)–(8).

In the above model, J denotes the desirable objective of the ridesharing platform (to be minimized or maximized); Eqs. (1)–(8) define the DUE with ridesharing on a single bottleneck; $m_r(t)$ and $m_p(t)$ are the charging and compensation, which are the decision variables to be solved, depending on the specific objective the platform aims to optimize. In this paper, $m_r(t)$ and $m_p(t)$ are defined as linear functions with departure time t and their specific forms (based on the objective functions of the platform) will be given in Sections 3 and 4. Model (9) has a bi-level structure, with the lower level being the DCS-based DUE problem under a given charging-compensation scheme and the upper level to optimize for the VCS by the platform to achieve its desirable objective. The bi-level problem in (9) can be simplified as a single level problem in later sections of the paper since the charging-compensation scheme is imposed on a single bottleneck. Detailed analyses will be provided in Sections 3 and 4. In this way, the program in Eq. (9) differentiates the VCS in our paper with that of FCS in existing studies, which also shows the main problem we plan to solve in this research, i.e., to find the optimal VCS to achieve certain desired objective of the ridesharing platform under the DUE problem.

Next we investigate the properties of VCS when the lower level is at the user equilibrium.

At equilibrium, commuters choose the departure time and travel mode with minimal trip costs. For travelers departing from the origin at time $t(t \in T)$, if the ridership of ridesharing is positive, we have $c_r(t) = c_p(t)$ which implies the ridesharing drivers and passengers share the identical travel costs. This leads to

$$m_p(t) + m_r(t) = (f + h_r - h_p)\tau(t), \tag{10a}$$

and

$$m_r(t) \geq h_r\tau(t). \tag{10b}$$

Eq. (10b) is obtained from Eqs. (4) and (5), which guarantees commuters participating in ridesharing. Obviously if $m_r(t) < h_r\tau(t)$, there is always $c_s(t) < c_r(t)$, ridesharing drivers would prefer to drive alone on their commute. That is, Eq. (10b) is the necessary condition for drivers willing to share a trip with passengers.

Let T_s and T_r separately denote the departure time windows for solo drivers and ridesharing participants, we obtain $T = T_s \cup T_r$. For the DUE state on a single bottleneck, separately letting relations $\dot{c}_s(t) = 0$, $t \in T_s$ and $\dot{c}_r(t) = 0$, $t \in T_r$ hold, we obtain

$$\dot{\tau}(t) = \begin{cases} \dfrac{\beta}{\alpha - \beta + f}, t + \tau(t) < t^*, \\ \dfrac{-\gamma}{\alpha + \gamma + f}, t + \tau(t) > t^*, \end{cases} t \in T_s, \tag{11a}$$

$$\dot{\tau}(t) = \begin{cases} \dfrac{\beta + \dot{m}_r(t)}{\alpha - \beta + f + h_r}, t + \tau(t) < t^*, \\ \dfrac{-\gamma + \dot{m}_r(t)}{\alpha + \gamma + f + h_r}, t + \tau(t) > t^*, \end{cases} t \in T_r. \tag{11b}$$

Notice that the above general properties do not depend on the specific objective of the platform. Next we give some general expressions of different platform objectives. First we focus on the platform's profit. The profit is defined as the integral of the difference between the charge from passengers and the compensation to ridesharing drivers for the entire study period:

$$\pi = \int_{t \in T_r} (m_p(t) - m_r(t))u_r(t)\mathrm{d}t. \tag{12}$$

Second, the system disutility is defined as the difference between the total travel cost of commuters and the profit of the ridesharing platform as follows:

$$G = N\widehat{c} - \pi. \tag{13}$$

In the following, the optimal VCS under different objectives will be investigated in Section 3; in Section 4, we will examine a special case when ridesharing participants commute with no queue.

3. Ridesharing with VCS

In this section, ridesharing with VCS is introduced under the model setting defined in Section 2. We will examine the optimal VCS under different objectives of the platform: (1) minimizing the system disutility; (2) maximizing the platform's profit; (3) minimizing the system disutility with zero profit of the platform.

For further analysis, we first give two propositions.

Proposition 1. *When the fuel cost parameter is larger than the summation of the inconvenience cost parameters for ridesharing drivers and passengers,* i.e., $f > h_r + h_p$, *to minimize the system disutility with VCS, all commuters should be involved in ridesharing.*

Proof. At equilibrium, for solo drivers and ridesharing participants separately departing at T_s and T_r, with Eqs. (4), (6) and (13), the system disutility can be expressed as

$$\begin{aligned} G &= N\widehat{c} - \pi \\ &= \int_{t \in T_s} c_s(t)u_s(t)\mathrm{d}t + \int_{t \in T_r} (c_r(t) + c_p(t))u_r(t)\mathrm{d}t - \int_{t \in T_r} (m_p(t) - m_r(t))u_r(t)\mathrm{d}t \\ &= \int_{t \in T_s} c_s(t)u_s(t)\mathrm{d}t + \int_{t \in T_r} (2c_s(t) + (h_r + h_p - f)\tau(t))u_r(t)\mathrm{d}t. \end{aligned}$$

The above relation holds for any T_s, $T_r \geq 0$, $T_s \cup T_r = T$. Selecting two solo drivers departing at $t'(t' \in T_s)$ and letting them choose ridesharing such that one of them would be the ridesharing driver and the other one the passenger, for any types of charging-compensation scheme satisfying $m_r(t) \geq h_r\tau(t)$. Forcing the ridesharing driver of that pair to depart at t', then the corresponding system disutility, after the solo drivers become the ridesharing pair, can be expressed as

$$\begin{aligned}G' &= \int_{t\in T_s} c_s(t)u_s(t)\mathrm{d}t - 2c_s\left(t'\right) + \left(2c_s\left(t'\right) + (h_r + h_p - f)\tau\left(t'\right)\right) + \int_{t\in T_r} (2c_s(t) + (h_r + h_p - f)\tau(t))u_r(t)\mathrm{d}t \\ &= \int_{t\in T_s} c_s(t)u_s(t)\mathrm{d}t + \int_{t\in T_r} (2c_s(t) + (h_r + h_p - f)\tau(t))u_r(t)\mathrm{d}t + (h_r + h_p - f)\tau\left(t'\right) \geq G'',\end{aligned}$$

where G'' is the equilibrium system disutility satisfying $G' \geq G''$ since at equilibrium, drivers will choose the departure time with minimal travel cost. Compared to G, we find there is $G - G' = (f - h_r - h_p)\tau(t') > 0$ (induced by $f > h_r + h_p$), leading to relation $G \geq G''$ holds. That is, the system disutility is lesser with more commuters involved in ridesharing. Extending above results, we conclude that when the ridesharing platform aims to minimize the system disutility with VCS, all commuters should be involved in ridesharing. This completes the proof. □

Proposition 2. *When the fuel cost parameter is larger than the summation of the inconvenience cost parameters for ridesharing drivers and passengers,* i.e., $f > h_r + h_p$, *to maximize the platform's profit with VCS, all commuters should be involved in ridesharing. Furthermore, the optimal VCS has to follow this form:* $m_r(t) = h_r\tau(t)$ *and* $m_p(t) = (f - h_p)\tau(t)$.

Proof. With Eqs. (10), (12), we have

$$\begin{aligned}\pi &= \int_{t\in T_r} (m_p(t) - m_r(t))u_r(t)\mathrm{d}t \\ &= \int_{t\in T_r} ((f + h_r - h_p)\tau(t) - 2m_r(t))u_r(t)\mathrm{d}t \\ &\leq \int_{t\in T_r} ((f - h_r - h_p)\tau(t))u_r(t)\mathrm{d}t.\end{aligned} \tag{14}$$

The equality holds only when $m_r(t) = h_r\tau(t)$ (Eq. (10b)). In this way, the platform compensates each ridesharing driver with $h_r\tau(t)$ and charges each passenger with $(f - h_p)\tau(t)$, and the profit is computed as $\pi = \int_{t\in T_r} ((f - h_r - h_p)\tau(t))u_r(t)\mathrm{d}t$.

When there is $f - h_r - h_p > 0$, with Eq. (14), let $m_r(t) = h_r\tau(t)$ and $m_p(t) = (f - h_p)\tau(t)$, we conclude that

$$\int_{t\in T_r} ((f - h_r - h_p)\tau(t))u_r(t)\mathrm{d}t \leq \int_{t\in T} ((f - h_r - h_p)\tau(t))u(t)\mathrm{d}t.$$

We can prove that the equality holds only when all commuters are involved in ridesharing. To see this, assuming there are solo drivers when the profit is maximized. Select two solo drivers departing at $t'(t' \in T_s)$. If they form the ridesharing pair, the compensation to the driver is $h_r\tau(t')$ and the charge to the passenger is $(f - h_p)\tau(t')$. Then the platform's profit, after the solo drivers become the ridesharing pair, can be explicitly expressed as

$$\pi' = \int_{t\in T_r} ((f - h_r - h_p)\tau(t))u_r(t)\mathrm{d}t + (f - h_r - h_p)\tau\left(t'\right),$$

which is larger than the earlier profit $\pi = \int_{t\in T_r} ((f - h_r - h_p)\tau(t))u_r(t)\mathrm{d}t$ if we assume $f - h_r - h_p > 0$. That is, the platform's profit is increased after the solo drivers departing at $t'(t' \in T_s)$ are converted to ridesharing pair. Therefore, with $m_r(t) = h_r\tau(t)$ and $m_p(t) = (f - h_p)\tau(t)$, extending the results obtained above to the whole study period, we find that the profit is maximized only when all commuters are involved in ridesharing. We complete the proof. □

In Eq. (14) with $\tau(t)$, $u(t) \geq 0$, if $f \leq h_r + h_p$, we get $\pi \leq 0$. That is, the ridesharing platform always makes non-positive profit with VCS if the fuel cost parameter is less than the summation of the inconvenience cost parameters for the ridesharing drivers and the passengers. If there is $f - h_r - h_p > 0$, the platform's profit can be maximized with VCS following the form: $m_r(t) = h_r\tau(t)$ and $m_p(t) = (f - h_p)\tau(t)$.

With above analyses, we give the remark as follows.

Remark 1. The fuel cost and the inconvenience cost parameters for ridesharing drivers and passengers play a vital role in the analyses, and the case when the sum of the inconvenience cost parameters for ridesharing drivers and passengers is smaller than the fuel cost parameter is worth to be further investigated.

The condition of $f - h_r - h_p > 0$ is always assumed hereafter in this paper. As stated in Proposition 1 and Proposition 2, if the ridesharing platform aims to minimize the system disutility or maximize the profit, all commuters should be involved in ridesharing. Now suppose that the system disutility is minimized with platform's zero profit, if there exist solo drivers, then let them participate in ridesharing under FCS (a special case of VCS), the system disutility can be further reduced (see Proposition 1). That is, when the platform aims to minimize the system disutility with zero profit, there should be no solo drivers. As a result, the user equilibria under three objectives are investigated when all commuters are involved in the ridesharing. In this way, there should be $N_s = 0$, $N_r = N/2$; $T_s = \emptyset$, $T_r = T$.

3.1. Platform minimizes the system disutility

With Proposition 1, when the ridesharing platform aims to minimize the system disutility, we rewrite the formulation in Eq. (9) as below:

$$\begin{aligned}\min_{m_r(t),m_p(t)} G &= \int_{t\in T}(c_r(t)+c_p(t))u(t)\mathrm{d}t - \int_{t\in T}(m_p(t)-m_r(t))u(t)\mathrm{d}t \\ &= \int_{t\in T}2(\max\{\beta(t^*-t-\tau(t)),\gamma(t+\tau(t)-t^*)\})u(t)\mathrm{d}t + \int_{t\in T}2\Big(\alpha\tau(t)+\frac{1}{2}(h_r+h_p+f)\tau(t)\Big)u(t)\mathrm{d}t,\end{aligned} \tag{15}$$

subject to Eqs. (1)–(8).

To study the solutions of above program, we give the theorem as follows.

Theorem 1. *When the ridesharing platform aims to minimize the system disutility with VCS, the model exists unique solution. Under this case, the user equilibrium coincides with the system optimum.*

Proof. To obtain a solution of the program (15), we need to find the optimal $m_r(t)$ and $m_p(t)$, such that the objective function in (15) achieves its minimum. Model (15) is similar to find solutions satisfying the system optimum with road pricing in the traditional bottleneck model (Arnott et al., 1990a), where there is always $\tau(t)=\tau_0$, $\forall t\in T$. To see this, notice that the total schedule delay costs, i.e., $\int_{t\in T}2(\max\{\beta(t^*-t-\tau(t)),\ \gamma(t+\tau(t)-t^*)\})u(t)\mathrm{d}t$, is a constant at equilibrium (user equilibrium or system optimum) in the traditional bottleneck model. Furthermore, the remaining part related to the travel time, $\int_{t\in T}2(\alpha\tau(t)+\frac{1}{2}(h_r+h_p+f)\tau(t))u(t)\mathrm{d}t$, reaches the minimum when there is $\tau(t)=\tau_0$, $\forall t\in T$. Therefore, solutions with $\tau(t)=\tau_0$, $\forall t\in T$, can solve the program in Eq. (15), indicating that the solutions of user equilibrium lead to the system optimum. Under this case, VCS achieves minimal system disutility without road pricing.

At system optimum in the traditional bottleneck model, there should be

$$\begin{cases}\beta(t^*-t_1-\tau_0)=\gamma(t_3+\tau_0-t^*),\\ (t_3+\tau_0)-(t_1+\tau_0)=\dfrac{1}{2}N/s,\end{cases}$$

where t_1 and t_3 is respectively the earliest and latest departure time. The last relation holds since there is $N/2$ number of vehicular traffic.

Since all commuters are subject to identical travel times, for drivers departing at time t_2 with $t_2+\tau_0=t^*$, the earliest, critical and latest departure times are derived as

$$\begin{cases}t_1=t^*-\tau_0-\dfrac{\gamma}{\beta+\gamma}\dfrac{N}{2s},\\ t_2=t^*-\tau_0,\\ t_3=t^*-\tau_0+\dfrac{\beta}{\beta+\gamma}\dfrac{N}{2s}.\end{cases} \tag{16}$$

With Eq. (16), we find the equilibrium can be uniquely determined and the solution uniquely exists. We complete the proof. □

To find the corresponding $m_r(t)$ and $m_p(t)$, letting $\dot{c}_r(t)=0$ in Eq. (11b), there is $\dot{m}_r(t)=\{\begin{smallmatrix}-\beta,\, t+\tau_0<t^*\\ \gamma,\, t+\tau_0>t^*\end{smallmatrix}$. That is, the compensation function for ridesharing drivers is piecewise linear. Letting

$$m_r(t)=\begin{cases}\beta(t^*-t-\tau_0)+\phi_1,\, t+\tau_0\le t^*,\\ \gamma(t+\tau_0-t^*)+\phi_1,\, t+\tau_0\ge t^*,\end{cases} \tag{17}$$

where ϕ_1 is a constant with $\phi_1\ge h_r\tau_0$, this leads to $m_r(t)\ge h_r\tau_0$. Together with (10b), we find all travelers would prefer to participate in ridesharing. Eq. (17) states the compensation function first decreases with slope β then increases with γ, with the minimal compensation value at the critical time.

The identical generalized travel cost for each commuter is $\widehat{c}=(\alpha+f+h_r)\tau_0-\phi_1$.

In Eq. (17), letting $\phi_1=h_r\tau_0$. With Eqs. (16) and (17), when the ridesharing platform aims to minimize the system disutility, the optimal VCS is

$$m_r(t)=\begin{cases}\beta(t^*-t-\tau_0)+h_r\tau_0,\, t\in\left[t^*-\tau_0-\dfrac{\gamma}{\beta+\gamma}\dfrac{N}{2s},\, t^*-\tau_0\right],\\ \gamma(t+\tau_0-t^*)+h_r\tau_0,\, t\in\left[t^*-\tau_0,\, t^*-\tau_0+\dfrac{\beta}{\beta+\gamma}\dfrac{N}{2s}\right],\end{cases} \tag{18a}$$

$$m_p(t) = \begin{cases} (f-h_p)\tau_0 - \beta(t^* - t - \tau_0), t \in \left[t^* - \tau_0 - \dfrac{\gamma}{\beta+\gamma}\dfrac{N}{2s}, t^* - \tau_0\right], \\ (f-h_p)\tau_0 - \gamma(t + \tau_0 - t^*), t \in \left[t^* - \tau_0, t^* - \tau_0 + \dfrac{\beta}{\beta+\gamma}\dfrac{N}{2s}\right]. \end{cases} \tag{18b}$$

With above analyses, we find the charging-compensation scheme uniquely exists, and the charges behaves oppositely compared to that of the compensation function.

Now we have obtained the optimal VCS and the solutions of user equilibrium when the platform minimizes the system disutility. Under this case, the following theorem provides some properties of the platform's profit.

Theorem 2. *When the platform minimizes the system disutility, the correspondingly platform's profit largely depends on the default parameters h_r, h_p, f, β, γ, τ_0, s and N. If the relation $h_r + h_p < f - \frac{N\beta\gamma}{2(\beta+\gamma)\tau_0 s}$ holds, the platform makes positive profit; conversely, the profit is non-positive.*

Proof. With Eqs. (12), (16) and (17), the profit of the platform is computed as

$$\begin{aligned} \pi &= \int_{t_1}^{t_3} (m_p(t) - m_r(t))u(t)\mathrm{d}t \\ &= s\int_{t_1}^{t_2} ((f+h_r-h_p)\tau_0 - 2\beta(t^* - t - \tau_0))\mathrm{d}t + s\int_{t_2}^{t_3} ((f+h_r-h_p)\tau_0 - 2\gamma(t+\tau_0-t^*))\mathrm{d}t - 2\phi_1(t_3 - t_1)s \\ &= -\frac{\beta\gamma}{4(\beta+\gamma)}\frac{N^2}{s} + \frac{N}{2}(f+h_r-h_p)\tau_0 - N\phi_1. \end{aligned}$$

In Eq. (17), since there is $\phi_1 \geq h_r\tau_0$, let $\phi_1 = h_r\tau_0$, the platform's profit in above relation reaches the maximum with $\pi = -\frac{\beta\gamma}{4(\beta+\gamma)}\frac{N^2}{s} + \frac{N}{2}(f - h_r - h_p)\tau_0$. For predefined parameters (N, f, τ_0, β, γ, s), letting the profit function larger than zero, there should be

$$h_r + h_p < f - \frac{N\beta\gamma}{2(\beta+\gamma)\tau_0 s}. \tag{19}$$

Namely, if the sum of inconvenience cost parameters for both ridesharing drivers and passengers is smaller than the threshold value in the right hand side of (19), the platform makes positive profit; otherwise, the profit is always non-positive. We complete the proof. □

With Theorem 2, if Eq. (19) is satisfied, the platform makes positive profit when the system disutility is also at the minimum. Note that the right hand side in Eq. (19) is linear with the travel demand N. For ridesharing cases with high travel demand (e.g., large cities or high demand for a studied corridor), that relation can be difficult to be satisfied due to the potentially large N. It implies that to reach the minimum system disutility in these cases, the above relation may not hold. As a result, the platform has to make subsidies to the ridesharing system (i.e., negative profit) in order to minimize the system disutility. However, a platform's negative profit is usually impossible to have a sustainable operation of the platform in a competitive market, unless there is continuous funding from the city authority. Therefore, other VCS that can lead to non-negative profit of the platform are worth of investigation. The following analyses assume $f - \frac{N\beta\gamma}{2(\beta+\gamma)\tau_0 s} \leq h_r + h_p < f$, i.e., the platform makes non-positive profit when the system disutility is at the minimum.

With the equilibrium travel cost and the platform's profit, the system disutility can be expressed as

$$G = N\hat{c} - \pi = N\left(\frac{\beta\gamma}{4(\beta+\gamma)}\frac{N}{s} + \left(\alpha + \frac{1}{2}(f+h_r+h_p)\right)\tau_0\right).$$

According to Theorem 1, we conclude that compared to the system optimum in the traditional bottleneck model, the travel cost for each traveler is lower with VCS due to the reduced vehicular traffic with ridesharing; furthermore, the resulting minimal system disutility is lesser.

Until now, we have examined the case when the platform aims to minimize the system disutility. With VCS following the form in Eq. (18), the system optimum can be obtained without imposing any pricing policies, which is of great importance since in the traditional bottleneck model, the system optimum can only be obtained by the adoption of road use pricing. In this way, we provide a new perspective for the ridesharing platform to achieve system optimum by designing flexible charging-compensation schemes.

3.2. Platform maximizes the profit

Under the model setting in Section 2, with Proposition 2, when the platform maximizes the profit, the problem can be formulated as

$$\max_{m_r(t), m_p(t)} \pi = \int_{t\in T} (m_p(t) - m_r(t))u(t)\mathrm{d}t$$

$$= \int_{t\in T} ((f + h_r - h_p)\tau(t) - 2m_r(t))u(t)\mathrm{d}t,$$

subject to Eqs. (1)–(8).

To find the optimal solutions satisfying the above program, we give the theorem as follows.

Theorem 3. *When the ridesharing platform aims to maximize its profit with VCS, the solution exists. Furthermore, the optimal VCS has to follow this form:* $m_r(t) = h_r\tau(t)$ *and* $m_p(t) = (f - h_p)\tau(t)$.

Proof. The proof of this can be totally obtained from Proposition 2, in which the platform should charge each passenger with $(f - h_p)\tau(t)$ and compensate each driver with $h_r\tau(t)$. Under this case, the profit is maximized with $\int_{t\in T} ((f - h_r - h_p)\tau(t))u(t)\mathrm{d}t$. Once the charging-compensation scheme is determined, the ridesharing system will reach the user equilibrium as described in Section 2.6. □

Let $m_r(t) = h_r\tau(t)$ and $m_p(t) = (f - h_p)\tau(t)$, then the charging-compensation functions are piecewise linear with time t since the travel time function $\tau(t)$ is piecewise linear with time in the traditional bottleneck model for the morning commute problems. Substituting relations $m_r(t)$ and $m_p(t)$ into Eq. (11b), there is

$$\dot{\tau}(t) = \begin{cases} \dfrac{\beta}{\alpha - \beta + f}, t + \tau(t) < t^*, \\ \dfrac{-\gamma}{\alpha + \gamma + f}, t + \tau(t) > t^*. \end{cases}$$

That is, the change rate of travel time is equal to that without ridesharing.

For ridesharing drivers departing at the earliest and latest times, there should be

$$\begin{cases} (\alpha + f)\tau_0 + \beta(t^* - t_1 - \tau_0) = (\alpha + f)\tau_0 + \gamma(t_3 + \tau_0 - t^*), \\ (t_3 + \tau_0) - (t_1 + \tau_0) = \dfrac{1}{2}N/s. \end{cases}$$

With $t_2 + \tau(t_2) = t^*$, where $\tau(t)$ is defined in Eq. (3), the earliest, critical and latest departure times are derived as

$$\begin{cases} t_1 = t^* - \tau_0 - \dfrac{\gamma}{\beta + \gamma}\dfrac{N}{2s}, \\ t_2 = t^* - \tau_0 - \dfrac{\beta\gamma}{(\alpha + f)(\beta + \gamma)}\dfrac{N}{2s}, \\ t_3 = t^* - \tau_0 + \dfrac{\beta}{\beta + \gamma}\dfrac{N}{2s}. \end{cases} \tag{20}$$

According to Theorem 3, with the change rate of travel time and the departure times in Eq. (20), the optimal VCS when the ridesharing platform aims to maximize its profit is

$$m_r(t) = \begin{cases} \dfrac{h_r\beta}{\alpha - \beta + f}\left(t + \tau_0 + \dfrac{\gamma}{\beta + \gamma}\dfrac{N}{2s} - t^*\right) + h_r\tau_0, t \in [t_1, t_2], \\ \dfrac{-h_r\gamma}{\alpha + \gamma + f}\left(t + \tau_0 - \dfrac{\beta}{\beta + \gamma}\dfrac{N}{2s} - t^*\right) + h_r\tau_0, t \in [t_2, t_3], \end{cases} \tag{21a}$$

$$m_p(t) = \begin{cases} \dfrac{(f - h_p)\beta}{\alpha - \beta + f}\left(t + \tau_0 + \dfrac{\gamma}{\beta + \gamma}\dfrac{N}{2s} - t^*\right) + (f - h_p)\tau_0, t \in [t_1, t_2], \\ \dfrac{-(f - h_p)\gamma}{\alpha + \gamma + f}\left(t + \tau_0 - \dfrac{\beta}{\beta + \gamma}\dfrac{N}{2s} - t^*\right) + (f - h_p)\tau_0, t \in [t_2, t_3]. \end{cases} \tag{21b}$$

where t_1, t_2 and t_3 are given in Eq. (20).

Since there is $\alpha > \beta$, the compensation function first increases for early arrivals and then decreases for late arrivals, indicating that the compensation reaches the maximum at the critical time. Although the tendency of charging function is similar to that of compensation, with relation $f - h_r - h_p > 0$, we conclude that the charging function changes dramatically than the compensation.

The generalized travel cost for each commuter is $\widehat{c} = \frac{\beta\gamma}{\beta+\gamma}\frac{N}{2s} + (\alpha + f)\tau_0$.

Recalling Theorem 3, the platform's maximum profit can be expressed

$$\begin{aligned} \pi &= \int_{t\in T} ((f - h_r - h_p)\tau(t))u(t)\mathrm{d}t. \\ &= (f - h_r - h_p)\left(\frac{N}{2}\tau_0 + \frac{\beta\gamma}{(\alpha + f)(\beta + \gamma)}\frac{N^2}{8s}\right). \end{aligned} \tag{22}$$

(See Appendix A)

With the above analyses, we conclude that the platform's profit is positive when the platform aims to maximize it. In this case, the system disutility can be computed as

$$G = N\widehat{c} - \pi = \frac{\beta\gamma}{\beta+\gamma}\frac{N^2}{2s} - (f - h_r - h_p)\left(\frac{\beta\gamma}{(\alpha+f)(\beta+\gamma)}\frac{N^2}{8s}\right) + N\left(\alpha + \frac{1}{2}(f + h_r + h_p)\right)\tau_0.$$

3.3. Platform minimizes the system disutility with zero profit

As stated in Section 2, for simplicity and without loss of generality, the linear charging-compensation function with departure time t is assumed. Let t_1, t_2 and t_3 respectively be the earliest, critical and latest departure times of travelers. Since the travel time function behaves linearly with time t for early and late arrivals, let the compensation function to the ridesharing drivers be:

$$m_r(t) = \begin{cases} k_1(t^* - t - \tau(t)) + \phi_2, t + \tau(t) \le t^*, \\ k_2(t + \tau(t) - t^*) + \phi_2, t + \tau(t) \ge t^*, \end{cases} \tag{23}$$

where k_1 and k_2 are the coefficients integrating the delay of drivers respectively for early and late arrivals with the time-varying compensation, ϕ_2 is a constant guaranteeing the continuity of the compensation function at the critical departure time t_2. In Eq. (23), k_1, k_2 and ϕ_2 are decision variables to be solved under the model setting. In this way, the function $m_r(t)$ is piecewise linear with two linear pieces for early and late arrivals.

Substituting the compensation function (23) into (11b), we obtain the change rate of the travel time and the departure rate of vehicular traffic as follows:

$$\dot{\tau}(t) = \begin{cases} \dfrac{\beta - k_1}{\alpha + f + h_r - \beta + k_1}, t_1 < t < t_2, \\ \dfrac{k_2 - \gamma}{\alpha + f + h_r + \gamma - k_2}, t_2 < t < t_3, \end{cases} \tag{24a}$$

$$u(t) = (1 + \dot{\tau}(t))s = \begin{cases} \dfrac{\alpha + f + h_r}{\alpha + f + h_r - \beta + k_1}s, t_1 < t < t_2, \\ \dfrac{\alpha + f + h_r}{\alpha + f + h_r + \gamma - k_2}s, t_2 < t < t_3. \end{cases} \tag{24b}$$

Given the change rate of travel time in Eq. (24a), the travel time for travelers departing at time t can be formulated as a continuous piecewise function:

$$\tau(t) = \begin{cases} \dfrac{\beta - k_1}{\alpha + f + h_r - \beta + k_1}(t - t_1) + \tau_0, t_1 \le t \le t_2, \\ \dfrac{k_2 - \gamma}{\alpha + f + h_r + \gamma - k_2}(t - t_3) + \tau_0, t_2 \le t \le t_3. \end{cases} \tag{25}$$

The equilibrium conditions that the ridesharing drivers have identical travel cost at t_1 and t_3 give

$$\begin{cases} (\alpha + f + h_r)\tau_0 + (\beta - k_1)(t^* - t_1 - \tau_0) = (\alpha + f + h_r)\tau_0 + (\gamma - k_2)(t_3 + \tau_0 - t^*), \\ \dfrac{\alpha + f + h_r}{\alpha + f + h_r - \beta + k_1}s(t_2 - t_1) + \dfrac{\alpha + f + h_r}{\alpha + f + h_r + \gamma - k_2}s(t_3 - t_2) = N/2. \end{cases} \tag{26}$$

Travelers departing at time t_2 arrive at the destination punctually. It follows

$$t_2 + \frac{\beta - k_1}{\alpha + f + h_r - \beta + k_1}(t_2 - t_1) + \tau_0 = t^*. \tag{27}$$

Given Eq. (25), the continuity of the travel time function requires that at time t_2, there is

$$\frac{\beta - k_1}{\alpha + f + h_r - \beta + k_1}(t_2 - t_1) = \frac{k_2 - \gamma}{\alpha + f + h_r + \gamma - k_2}(t_2 - t_3). \tag{28}$$

With above analyses, we can obtain

$$\begin{cases} t_1 = t^* - \tau_0 - \dfrac{\gamma}{\beta+\gamma}\dfrac{N}{2s}, \\ t_2 = t^* - \tau_0 - \dfrac{\beta - k_1}{\alpha + f + h_r}\dfrac{\gamma}{\beta+\gamma}\dfrac{N}{2s}, \\ t_3 = t^* - \tau_0 + \dfrac{\beta}{\beta+\gamma}\dfrac{N}{2s}, \end{cases} \tag{29a}$$

$$\frac{\beta-k_1}{\alpha+f+h_r-\beta+k_1}\gamma\left(1-\frac{\beta-k_1}{\alpha+f+h_r}\right)=\frac{k_2-\gamma}{\alpha+f+h_r+\gamma-k_2}\left(-\frac{\beta-k_1}{\alpha+f+h_r}\gamma-\beta\right). \tag{29b}$$

At equilibrium, for early arrivals, the departure rate is larger than the bottleneck capacity; while for late arrivals, the departure late is smaller than the bottleneck capacity, with both rates being positive. Together with the necessary conditions in Eq. (10b) which guarantees that commuters participating in ridesharing, there should be

$$u(t)\Big|_{t\in(t_1,t_2)}\geq s\geq u(t)\Big|_{t\in(t_2,t_3)}>0;\ m_r(t)\geq h_r\tau(t),\ \forall t_1\leq t\leq t_3. \tag{30}$$

That is, to obtain the user equilibrium, the requirements in Eq. (30) should be satisfied.
Substituting (23), 24b) and (25) into above relations, we obtain the equivalent statement

$$\begin{cases}\beta-(\alpha+f+h_r)\leq k_1\leq\beta,\\ k_2\leq\gamma,\\ h_r\tau_0\leq k_1(t^*-t_1-\tau_0)+\phi_2,\\ h_r\tau_0\leq\phi_2,\\ h_r\tau_0\leq k_2(t_3+\tau_0-t^*)+\phi_2.\end{cases} \tag{31}$$

At equilibrium, the generalized travel cost is

$$\widehat{c}=(\alpha+f+h_r)\tau_0+(\beta-k_1)(t^*-t_1-\tau_0)-\phi_2. \tag{32}$$

With above relations, the platform's profit can be derived as

$$\pi=\int_{t_1}^{t_3}((f+h_r-h_p)\tau(t)-2m_r(t))u(t)\mathrm{d}t, \tag{33}$$

where $m_r(t)$, $u(t)$, $\tau(t)$, t_1, t_2, t_3 is separately given in (23), (24b), (25), (29a).

Now we have established the user equilibrium given the parameters k_1, k_2 and ϕ_2, and the corresponding results of t_1, t_2 and t_3 can be obtained in (29a). Under the model setting in Section 2, to find the optimal VCS when the platform minimizes the system disutility with zero profit, we can write the problem as below:

$$\min_{k_1,k_2,\phi_2} G=N((\alpha+f+h_r)\tau_0+(\beta-k_1)(t^*-t_1-\tau_0)-\phi_2), \tag{34a}$$

subject to (29b), (31) and

$$\pi=0, \tag{34b}$$

$$0\leq t_1\leq t_2\leq t_3, \tag{34c}$$

where t_1, t_2 and t_3 are defined in (29a). In the above program, Eq. (34a) gives the system disutility (since the platform's profit is zero); Eqs. (29b) and (31) are derived from the user equilibrium; Eq. (34b) guarantees the zero-value profit of the platform and Eq. (34c) defines the bounds of the departure times.

Now we study the solutions of the optimization problem (34).

Theorem 4. *The optimization problem* (34) has at least one solution.

Proof. With the continuous objective function (34a), bounded and closed nonempty feasible set (e.g., solution with $m_r(t)=m_p(t)$ is feasible) constituted by constraints (29b), (31), (34b), (34c), there certainly exist solutions for the optimization problem (34). □

In the formulation (34), although there exist solutions satisfying the program, the uniqueness cannot be satisfied. Notice that there are only three decision variables of k_1, k_2 and ϕ_2, hence to solve the program is not that hard in spite of the strict restriction of $\pi=0$. Compared to Sections 3.1 and 3.2, the program in Section 3.3 is more complex and it is difficult to obtain analytical solutions directly. However, once k_1, k_2 and ϕ_2 are derived, we can obtain the optimal VCS and the corresponding solutions for the user equilibrium when the ridesharing platform minimizes the system disutility with zero profit.

By comparing Sections 3.2 and 3.3 to 3.1, we get the following remark.

Remark 2. Under the platform's objectives (2) and (3), there is $\tau(t)\neq\tau_0$, $\forall t\in(t_1,t_3)$.

Remark 2 is easier to be noticed since the solution with $\tau(t)=\tau_0$, $\forall t\in[t_1,t_3]$, is only obtained under objective (1), i.e., when the platform minimizes the system disutility. Remark 2 indicates that all ridesharing participants commute with a queue under the platform's objectives (2) and (3). Considering this, following section will be focused on a special case when the ridesharing participants commute with no queue.

4. Ridesharing participants commute with no queue

In this section, we examine the case when all ridesharing participants commute with no queue. Section 3 states that all travelers should be involved in ridesharing under the platform's objective of (1), (2) or (3). Furthermore, in Section 3.1, it shows that when objective (1) (i.e., minimizing the system disutility) is considered, in the optimal solution, all ridesharing participants will not experience any queue. Since this section focuses on the cases when ridesharing participants commute with no queue, objective (1) is already discussed in Section 3.1. Therefore only objectives (2) and (3) are investigated in this section, i.e. maximizing the platform's profit and minimizing the system disutility with zero profit of the ridesharing platform.

As revealed in Remark 2, under the platform's objectives of (2) and (3), when all commuters participate in ridesharing (discussed in Sections 3.2 and 3.3), they will experience a queue, except those who depart at the earliest or latest within the departure window. Therefore, if we want to examine the platform's objectives of (2) and (3) while keeping ridesharing participants commuting without queuing, some commuters should be solo drivers for their commute. We conclude that the major difference between Sections 3 and 4 is that there should have $N_s > 0$ for the cases studied in this section.

It should also be noted that if solo drivers depart at the two tails of departure time window and ridesharing participants depart in the middle of the window, then for early arrivals, the corresponding departure rate of solo drivers is lower than that of ridesharing participants, while for late arrivals, the departure rate of solo drivers is larger than that of ridesharing drivers (Arnott et al., 1994). This leads to all ridesharing commuters queuing at the bottleneck. Therefore, ridesharing participants should depart at the two tails of the departure time window since we aim to study the case when ridesharing participants commute without a queue, as shown in Fig. 1.

As defined in Section 2, N_s and N_r are used to denote the travel demands for solo drivers and ridesharing drivers, respectively, and the total vehicular traffic is $N_s + N_r$. Similar to the former analyses, t_1, t_2 and t_3 are used to depict the earliest, critical and latest departure times of drivers, respectively; t_4 and t_5 is the earliest and latest departure times of solo drivers, respectively, as shown in Fig. 1. In this case, the ridesharing participants commute with no queue and the queue is only experienced by the solo drivers.

At equilibrium, for solo drivers, there should be

$$\begin{cases} (\alpha+f)\tau_0 + \beta(t^* - t_4 - \tau_0) = (\alpha+f)\tau_0 + \gamma(t_5 + \tau_0 - t^*), \\ s(t_5 - t_4) = N_s. \end{cases} \tag{35}$$

Together with $t_2 + \tau(t_2) = t^*$, the solution of this problem is

$$\begin{cases} t_4 = t^* - \tau_0 - \dfrac{\gamma}{\beta+\gamma}\dfrac{N_s}{s}, \\ t_2 = t^* - \tau_0 - \dfrac{\beta\gamma}{(\alpha+f)(\beta+\gamma)}\dfrac{N_s}{s}, \\ t_5 = t^* - \tau_0 + \dfrac{\beta}{\beta+\gamma}\dfrac{N_s}{s}. \end{cases} \tag{36}$$

The generalized travel cost can be expressed as the function of N_s as below

$$\widehat{c} = \frac{\beta\gamma}{\beta+\gamma}\frac{N_s}{s} + (\alpha+f)\tau_0. \tag{37}$$

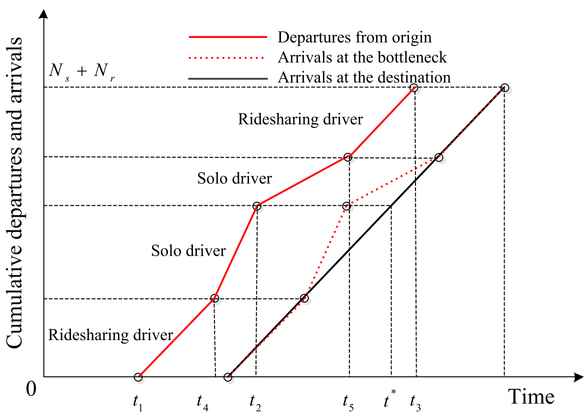

Fig. 1. Ridesharing participants commute with no queue.

Since there is $s(t_3-t_1)=N_s+N_r$, together with the relation $N_s+2N_r=N$, we obtain

$$t_3=\frac{1}{2s}(N+N_s)+t_1. \tag{38}$$

As shown in Fig. 1, with Eq. (11), the change rate of travel time is

$$\dot{\tau}(t)=\begin{cases}0, t\in(t_1,t_4)\cup(t_5,t_3),\\ \dfrac{\beta}{\alpha-\beta+f}, t\in(t_4,t_2),\\ \dfrac{-\gamma}{\alpha+\gamma+f}, t\in(t_2,t_5).\end{cases} \tag{39}$$

Therefore, we find there is $\dot{m}_r(t)=\{\begin{smallmatrix}-\beta, t\in(t_1,t_4)\\ \gamma, t\in(t_5,t_3)\end{smallmatrix}$. That is to say, the compensation function is piecewise linear with the first slope $-\beta$ and the other γ. Define the compensation function as

$$m_r(t)=\begin{cases}-\beta t+\phi_3, t\in[t_1,t_4],\\ \gamma t+\phi_4, t\in[t_5,t_3],\end{cases}$$

where ϕ_3 and ϕ_4 are constants to be solved. Since the travel costs among all travelers are identical, letting $c_r(t_4)=c_s(t_4)$, $c_r(t_5)=c_s(t_5)$, we get $m_r(t_4)=m_r(t_5)=h_r\tau_0$, $\phi_3=\beta(t^*-\tau_0)-\frac{\beta\gamma}{\beta+\gamma}\frac{N_s}{s}+h_r\tau_0$ and $\phi_4=-\gamma(t^*-\tau_0)-\frac{\beta\gamma}{\beta+\gamma}\frac{N_s}{s}+h_r\tau_0$. Therefore, there is $m_r(t)\geq h_r\tau_0$, which is in line with the necessary condition in Eq. (10b).

With above results, the compensation function paid to the ridesharing drivers can be expressed as

$$m_r(t)=\begin{cases}\beta(t^*-t-\tau_0)-\dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N_s}{s}+h_r\tau_0, t\in[t_1,t_4],\\ \gamma(t+\tau_0-t^*)-\dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N_s}{s}+h_r\tau_0, t\in[t_5,t_3].\end{cases} \tag{40}$$

Together with Eqs. (10) and (40), the profit function at time t is

$$m_p(t)-m_r(t)=\begin{cases}(f-h_r-h_p)\tau_0-2\beta(t^*-t-\tau_0)+\dfrac{2\beta\gamma}{\beta+\gamma}\dfrac{N_s}{s}, t\in[t_1,t_4],\\ (f-h_r-h_p)\tau_0-2\gamma(t+\tau_0-t^*)+\dfrac{2\beta\gamma}{\beta+\gamma}\dfrac{N_s}{s}, t\in[t_5,t_3].\end{cases} \tag{41}$$

There is $m_p(t_4)-m_r(t_4)=m_p(t_5)-m_r(t_5)=(f-h_r-h_p)\tau_0$.

Compared with the solutions in Section 3.1, there should be $t_1<t^*-\tau_0-\frac{\gamma}{\beta+\gamma}\frac{N}{2s}$ and $t_3>t^*-\tau_0+\frac{\beta}{\beta+\gamma}\frac{N}{2s}$, due to $N_s>0$. Together with the predefined relation $f-\frac{N\beta\gamma}{2(\beta+\gamma)\tau_0 s}\leq h_r+h_p<f$ (the conclusion of Theorem 2), we obtain

$$\begin{aligned}m_p(t_1)-m_r(t_1)&=m_p(t_3)-m_r(t_3)\\ &<(f-h_r-h_p)\tau_0-\frac{\beta\gamma}{\beta+\gamma}\frac{N}{s}+\frac{2\beta\gamma}{\beta+\gamma}\frac{N_s}{s}\\ &\leq\frac{\beta\gamma}{2(\beta+\gamma)s}(4N_s-N).\end{aligned} \tag{42}$$

For any $0<N_s\leq N/4$, there is always $m_p(t_1)-m_r(t_1)=m_p(t_3)-m_r(t_3)<0$. That is, with smaller N_s and larger N_r, the platform receives negative profit at the earliest and latest times.

From (41) and (42), we can find that the value $m_p(t)-m_r(t)$ increases for early arrivals and then decreases for late arrivals, together with the properly selected N_s, the curve of time-varying $m_p(t)-m_r(t)$ is shown in Fig. 2.

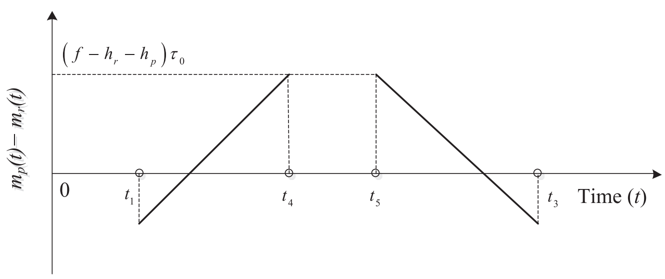

Fig. 2. The platform's profit function.

The following two subsections are conducted to find solutions of t_1, t_4, t_5, t_3, N_s and N_r satisfying two objectives of the platform: one is to maximize the platform's profit and the other is to minimize the system disutility with zero profit of the platform.

4.1. Platform maximizes the profit

In this subsection, we examine the appropriate VCS when the platform aims to maximize the profit under the model setting of Section 4.

Theorem 5. *When the platform aims to maximize the profit while keeping ridesharing participants commuting without a queue, there should be* $m_p(t_1) - m_r(t_1) = m_p(t_3) - m_r(t_3) = 0$.

Proof. As shown in Fig. 2, the profit function separately increases and decreases for early and late arrivals, with the minimum at t_1 and t_3. With Eq. (12), to maximize the profit, as much commuters as possible should be involved in ridesharing. However, as we concluded from Eq. (42), with larger N_r and smaller N_s, the profit is negative at the earliest and latest departure times. Therefore, for properly selected N_r and N_s such that relation $m_p(t_1) - m_r(t_1) = m_p(t_3) - m_r(t_3) = 0$ hold, there must be $m_p(t) - m_r(t) \geq 0$, $\forall t \in T$, then the platform's profit is maximized. □

With Theorem 5, substituting $m_p(t_1) - m_r(t_1) = m_p(t_3) - m_r(t_3) = 0$ into (41), we obtain

$$\begin{cases} t_1 = t^* - \tau_0 - \dfrac{1}{2\beta}(f - h_r - h_p)\tau_0 - \dfrac{\gamma}{\beta + \gamma}\dfrac{N_s}{s}, \\ t_3 = t^* - \tau_0 + \dfrac{1}{2\gamma}(f - h_r - h_p)\tau_0 + \dfrac{\beta}{\beta + \gamma}\dfrac{N_s}{s}. \end{cases} \tag{43}$$

Together with $N_s + 2N_r = N$ and $t_3 = \frac{1}{2s}(N + N_s) + t_1$ (Eq. (38)), there is

$$\begin{cases} N_s = N - \dfrac{\beta + \gamma}{\beta\gamma}(f - h_r - h_p)\tau_0 s, \\ N_r = \dfrac{\beta + \gamma}{2\beta\gamma}(f - h_r - h_p)\tau_0 s. \end{cases} \tag{44}$$

By substituting above relations into (36) and (43), the solutions of this problem can be computed as

$$\begin{cases} t_1 = t^* - \tau_0 - \dfrac{\gamma}{\beta + \gamma}\dfrac{N}{s} + \dfrac{\tau_0}{2\beta}(f - h_r - h_p), \\ t_4 = t^* - \tau_0 - \dfrac{\gamma}{\beta + \gamma}\dfrac{N}{s} + \dfrac{\tau_0}{\beta}(f - h_r - h_p), \\ t_2 = t^* - \tau_0 - \dfrac{\beta\gamma}{(\alpha + f)(\beta + \gamma)}\dfrac{N}{s} + \dfrac{\tau_0}{\alpha + f}(f - h_r - h_p), \\ t_5 = t^* - \tau_0 + \dfrac{\beta}{\beta + \gamma}\dfrac{N}{s} - \dfrac{\tau_0}{\gamma}(f - h_r - h_p), \\ t_3 = t^* - \tau_0 + \dfrac{\beta}{\beta + \gamma}\dfrac{N}{s} - \dfrac{\tau_0}{2\gamma}(f - h_r - h_p). \end{cases}$$

According to Eqs. (10), (40) and (44), the optimal VCS when the ridesharing platform aims to maximize its profit is

$$m_r(t) = \begin{cases} \beta(t^* - t - \tau_0) - \dfrac{\beta\gamma}{\beta + \gamma}\dfrac{N}{s} + (f - h_p)\tau_0, t \in [t_1, t_4], \\ \gamma(t + \tau_0 - t^*) - \dfrac{\beta\gamma}{\beta + \gamma}\dfrac{N}{s} + (f - h_p)\tau_0, t \in [t_5, t_3], \end{cases} \tag{45a}$$

$$m_p(t) = \begin{cases} \beta(t + \tau_0 - t^*) + \dfrac{\beta\gamma}{\beta + \gamma}\dfrac{N}{s} + h_r\tau_0, t \in [t_1, t_4], \\ \gamma(t^* - t - \tau_0) + \dfrac{\beta\gamma}{\beta + \gamma}\dfrac{N}{s} + h_r\tau_0, t \in [t_5, t_3]. \end{cases} \tag{45b}$$

With above analyses, we find the compensation function first decreases with slope β then increases with γ, while the charging function behaves oppositely. At the start and end of the study period, the compensations reach the maximum with $(f + h_r - h_p)\tau_0/2$, while the charging is at the minimum with $(f + h_r - h_p)\tau_0/2$. In this way, we find there is always $m_p(t) \geq m_r(t)$, $\forall t \in [t_1, t_4] \cup [t_5, t_3]$, which verifies the conclusion in Theorem 5.

Accordingly, the generalized travel cost can be computed as $\widehat{c} = \frac{\beta\gamma}{\beta + \gamma}\frac{N}{s} + (\alpha + h_r + h_p)\tau_0$.

To calculate the platform's profit, we need to compute the equation,

$$\pi = \int_{t_1}^{t_3} (m_p(t) - m_r(t))u_r(t)\mathrm{d}t = \frac{1}{4}\frac{\beta+\gamma}{\beta\gamma}((f - h_r - h_p)\tau_0)^2 s. \tag{46}$$

(See Appendix B)

Eq. (46) indicates that the platform's maximum profit is independent of the travel demand, and is just a function of the default parameters. This is different from what we have discussed in Section 3.2 where the profit function is quadratic with the commuting demand.

With the equilibrium travel cost and platform's profit, the system disutility can be computed as

$$G = N\left(\frac{\beta\gamma}{\beta+\gamma}\frac{N}{s} + (\alpha + h_r + h_p)\tau_0\right) - \frac{1}{4}\frac{\beta+\gamma}{\beta\gamma}((f - h_r - h_p)\tau_0)^2 s.$$

The duration of departure time window separately for solo drivers, ridesharing drivers, and the whole study period is

$$\begin{cases} |T_s| = t_5 - t_4 = \dfrac{N}{s} - \dfrac{(\beta+\gamma)\tau_0}{\beta\gamma}(f - h_r - h_p), \\ |T_r| = t_3 - t_1 - |T_s| = \dfrac{(\beta+\gamma)\tau_0}{2\beta\gamma}(f - h_r - h_p), \\ |T| = t_3 - t_1 = \dfrac{N}{s} - \dfrac{(\beta+\gamma)\tau_0}{2\beta\gamma}(f - h_r - h_p). \end{cases} \tag{47}$$

4.2. Platform minimizes the system disutility with zero profit

In this subsection, we study the appropriate VCS when the platform aims to minimize the system disutility with zero profit under the model setting of Section 4.

With Eq. (41), the platform's profit can be formulated as a function of t_1 and N_s,

$$\begin{aligned} \pi &= \int_{t_1}^{t_3} (m_p(t) - m_r(t))u_r(t)\mathrm{d}t \\ &= -(\beta+\gamma)(t^* - \tau_0 - t_1)^2 s + \gamma(N + N_s)(t^* - \tau_0 - t_1) \\ &\quad - \gamma\frac{(N+N_s)^2}{4s} + \frac{\beta\gamma}{\beta+\gamma}\frac{NN_s}{s} + \frac{N - N_s}{2}(f - h_r - h_p)\tau_0. \end{aligned} \tag{48}$$

(See Appendix C).

Note that the profit function is quadratic with t_1. Letting $\pi = 0$, we get

$$t_1 = t^* - \tau_0 - \frac{\gamma(N + N_s) \pm \sqrt{\Delta}}{2(\beta+\gamma)s}, \tag{49}$$

where

$$\Delta = -\beta\gamma(N - N_s)^2 + 2\tau_0 s(\beta+\gamma)(f - h_r - h_p)(N - N_s) \text{ (there should be } \Delta \geq 0). \tag{50}$$

That is, if relations (49) and (50) are satisfied, the platform's profit is zero. It is clear that t_1, as the earliest departure time, should be larger than zero and smaller than the earliest departure time of solo drivers, t_4 (given in Eq. (36)). We then get

$$t^* - \tau_0 - \frac{\gamma(N + N_s) \pm \sqrt{\Delta}}{2(\beta+\gamma)s} < t^* - \tau_0 - \frac{\gamma}{\beta+\gamma}\frac{N_s}{s}. \tag{51}$$

To find optimal VCS, we formulate the following optimization program,

$$\min_{N_s} G = N\left(\frac{\beta\gamma}{\beta+\gamma}\frac{N_s}{s} + (\alpha + f)\tau_0\right), \tag{52a}$$

subject to (49)–(51) and

$$0 < N_s < N. \tag{52b}$$

Note the generalized travel cost is just the function of N_s Eq. (37)). Hence the objective function in (52a) denotes the system disutility (the platform's profit is zero). Eqs. (49)–((51) guarantee the platform's zero profit and Eq. (52b) gives the bounds of N_s.

Now, we investigate some properties of this program.

Theorem 6. *The equilibrium solution of problem* (52) *exists and is unique.*

Proof. For any $N_s(0 < N_s < N)$ such that $\Delta \geq 0$, in Eq. (51), there is

$$\frac{\gamma(N+N_s) \pm \sqrt{\Delta}}{2(\beta+\gamma)s} - \frac{\gamma}{\beta+\gamma}\frac{N_s}{s} = \frac{\gamma(N-N_s) \pm \sqrt{\Delta}}{2(\beta+\gamma)s}.$$

Since $N_s < N$, if $\frac{\gamma(N-N_s)\pm\sqrt{\Delta}}{2(\beta+\gamma)s} > 0$, there should be $\gamma(N-N_s) > \sqrt{\Delta}$.
Substituting the values of Δ in Eq. (50) into above and let $\Delta \geq 0$, we obtain

$$N - \frac{2(\beta+\gamma)}{\beta\gamma}(f-h_r-h_p)\tau_0 s \leq N_s < N - \frac{2\tau_0 s}{\gamma}(f-h_r-h_p). \tag{53}$$

Notice that there is always $\frac{2(\beta+\gamma)}{\beta\gamma}(f-h_r-h_p)\tau_0 s > \frac{2\tau_0 s}{\gamma}(f-h_r-h_p)$, which is caused by the assumption that $\gamma > \beta$. Together with the linear objective function (52a), the whole programming (52) reaches the minimum at

$$N_s = N - \frac{2(\beta+\gamma)}{\beta\gamma}(f-h_r-h_p)\tau_0 s. \tag{54}$$

Substituting the value of N_s into (50), leads to $\Delta = 0$. Together with Eqs. (36), (38) and (49), the solution of this problem is

$$\begin{cases} t_1 = t^* - \tau_0 - \dfrac{\gamma}{\beta+\gamma}\dfrac{N}{s} + \dfrac{\tau_0}{\beta}(f-h_r-h_p), \\ t_4 = t^* - \tau_0 - \dfrac{\gamma}{\beta+\gamma}\dfrac{N}{s} + \dfrac{2\tau_0}{\beta}(f-h_r-h_p), \\ t_2 = t^* - \tau_0 - \dfrac{\beta\gamma}{(\alpha+f)(\beta+\gamma)}\dfrac{N}{s} + \dfrac{2\tau_0}{\alpha+f}(f-h_r-h_p), \\ t_5 = t^* - \tau_0 + \dfrac{\beta}{\beta+\gamma}\dfrac{N}{s} - \dfrac{2\tau_0}{\gamma}(f-h_r-h_p), \\ t_3 = t^* - \tau_0 + \dfrac{\beta}{\beta+\gamma}\dfrac{N}{s} - \dfrac{\tau_0}{\gamma}(f-h_r-h_p), \end{cases} \tag{55}$$

With above results, we get $t_1 < t_4 < t_2 < t_5 < t_3$. We complete the proof. □

According to Theorem 6, with Eqs. (9) and (40), the optimal VCS when the platform aims to minimize the system disutility with zero profit is

$$m_r(t) = \begin{cases} \beta(t^* - t - \tau_0) - \dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N}{s} + (2f - h_r - 2h_p)\tau_0, t \in [t_1, t_4], \\ \gamma(t + \tau_0 - t^*) - \dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N}{s} + (2f - h_r - 2h_p)\tau_0, t \in [t_5, t_3], \end{cases} \tag{56a}$$

$$m_p(t) = \begin{cases} \beta(t + \tau_0 - t^*) + \dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N}{s} - (f - 2h_r - h_p)\tau_0, t \in [t_1, t_4], \\ \gamma(t^* - t - \tau_0) + \dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N}{s} - (f - 2h_r - h_p)\tau_0, t \in [t_5, t_3]. \end{cases} \tag{56b}$$

Compared to VCS when the platform maximizes its profit, the charging-compensations in Eq. (56) present the same change rates. The compensations reach the maximum at the start and end of the departure time window, while the charges behave oppositely. In this way, we find there is $m_r(t_1) = m_r(t_3) = (f - h_p)\tau_0$, $m_r(t_4) = m_r(t_5) = h_r\tau_0$, $m_p(t_1) = m_p(t_3) = h_r\tau_0$, $m_p(t_4) = m_p(t_5) = (f - h_r)\tau_0$.

The corresponding generalized travel cost and the system disutility can be separately computed as $\widehat{c} = \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s} + (\alpha - f + 2h_r + 2h_p)\tau_0$ and $G = N(\frac{\beta\gamma}{\beta+\gamma}\frac{N}{s} + (\alpha - f + 2h_r + 2h_p)\tau_0)$.

With Theorem 6, the duration of the departure time window for solo drivers, ridesharing drivers, and the whole study period, respectively, is

$$\begin{cases} |T_s| = t_5 - t_4 = \dfrac{N}{s} - \dfrac{2(\beta+\gamma)\tau_0}{\beta\gamma}(f-h_r-h_p), \\ |T_r| = t_3 - t_1 - |T_s| = \dfrac{(\beta+\gamma)\tau_0}{\beta\gamma}(f-h_r-h_p), \\ |T| = t_3 - t_1 = \dfrac{N}{s} - \dfrac{(\beta+\gamma)\tau_0}{\beta\gamma}(f-h_r-h_p). \end{cases} \tag{57}$$

Comparing the results in Eqs. (44), (47), (54), (57), we get the remark as follows.

Table 3
Inputs for single bottleneck.

Parameter	Value	Unit
α	5	\$/h
β	3.05	\$/h
γ	11	\$/h
τ_0	0.5	h
t^*	8.5	am
s	600	veh/h
N	2000	veh
f	5	\$/h
h_r	0.2	\$/h
h_p	0.3	\$/h

Remark 3. When ridesharing participants commute with no queue, the number of ridesharing travelers should be smaller when the platform maximize the profit compared to that when the platform minimize the system disutility with zero profit, which also lead to the longer durations of departure time window for ridesharing drivers and whole study period.

5. Numerical examples

In this section, numerical examples are conducted to verify our analytical results. We also conduct sensitivity analysis to show how the results may vary with different parameter inputs. The LINGO software is used to solve the programming problems in Section 3.3 numerically.

The parameters adopted in this paper are mainly from Liu and Li (2017), where $\alpha=\$5$, $\beta=\$3.05$ and $\gamma=\$11$. The link free-flow travel time is assumed as $\tau_0=0.5$h. The desired arrival time for all the travelers is $t^*=8.5$h. The capacity of the bottleneck is $s=600$ veh/h and the total demand is $N=2000$ veh. The fuel cost parameter is \$5. The threshold value $f-N\beta\gamma/(2(\beta+\gamma)\tau_0 s)=-\$2.96$. Let the inconvenience cost parameter for ridesharing drivers and passengers separately be $h_r=\$0.2$ and $h_p=\$0.3$. In this way, the condition $f-\frac{N\beta\gamma}{2(\beta+\gamma)\tau_0 s}\le h_r+h_p<f$ holds. All the input parameters are listed in Table 3.

5.1. Ridesharing with VCS

When the ridesharing with VCS is applied, the user equilibrium with the minimum system disutility is shown in Fig. 3, where we find the travel time is constant among the whole study period. The congestion time window is [6.7, 8.4] with the duration 2.7 h. The flat curves of departure rate and travel time shown in Fig. 3(a) and (b) indicating that the user equilibrium coincides with the system optimum, which is certainly in line with the results analysed in Theorem 1. The curves of charging and compensation are portrayed in Fig. 3(c), where we find the charging curve increases first then decreases, while the compensation curve behaves in an opposite way. The charges (compensations) are equal to each other at the earliest and latest departure times. The system disutility is \$11,730, and the correspondingly platform's profit is -\$1730, suggesting that the platform has to subsidy the ridesharing drivers to reach the minimum system disutility, which verifies our conclusion obtained from Theorem 2.

When the platform maximizes the profit, the departure rate, travel time, charges and compensations are shown in Fig. 4. The departure rate in Fig. 4(a) shows there is 863 veh/h for early arrivals and 286 veh/h for late arrivals. In this way, the critical time is $t_2=7.6$h. The corresponding travel time patterns are portrayed in Fig. 4(b). The charging and compensation curves are given in Fig. 4(c), where we find the fees charged from ridesharing passengers are always larger than the compensations to drivers. For the earliest and latest departure times, we find there is $m_r(t)=0.1$ and $m_p(t)=2.4$, which can

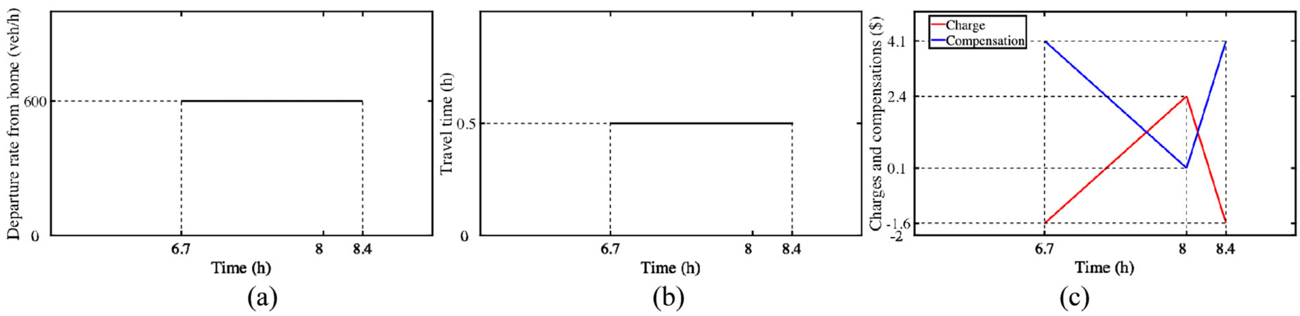

Fig. 3. Ridesharing with VCS when the platform minimizes the system disutility.

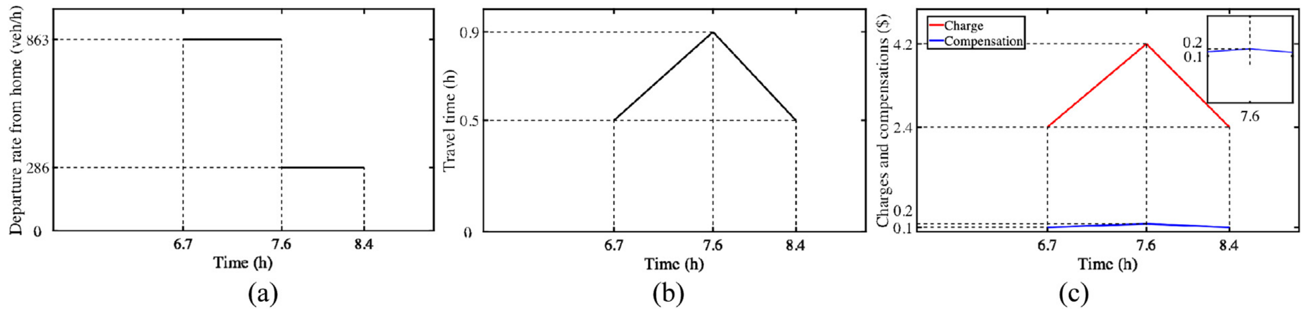

Fig. 4. Ridesharing with VCS when the platform maximizes the profit.

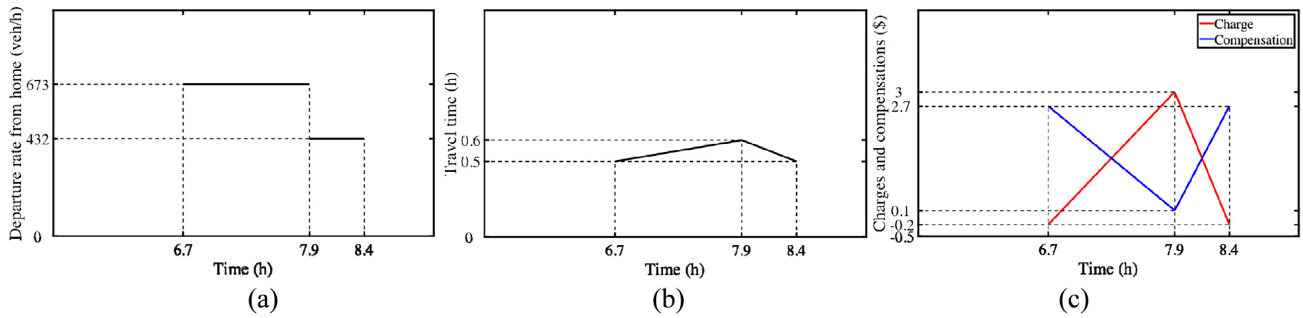

Fig. 5. Ridesharing with VCS when the platform minimizes the system disutility with zero profit.

be directly determined by Section 3.2. In this way, the platform's profit and the system disutility are \$3146 and \$14,814, respectively.

When the platform aims to minimize the system disutility with zero profit, we obtain the solution with $t_1 = 6.7$, $t_2 = 7.9$, $t_3 = 8.4$, $k_1 = 1.95$, $k_2 = 7.02$ and $\phi_2 = 0.13$. Substituting the solution into the charging-compensation function, we get $m_r(t) = -2.18t + 17.25$, $m_p(t) = 2.77t - 18.72$, $t \in [6.7, 7.9]$; $m_r(t) = 5.05t - 39.60$, $m_p(t) = -6.42t + 53.52$, $t \in [7.9, 8.4]$. The departure rate is potrayed in Fig. 5(a), where we find for early arrivals, the departure rate is smaller than that in Fig. 4(a), indicating that there is lesser queuing loss. The travel time patterns are portrayed in Fig. 5(b), and the corresponding charging and compensation curves are given in Fig. 5(c), where we find the patterns are similar to those in Fig. 3(c). The negative charging values at the two tails of the departure time window imply that the platform provides some subsidy to ridesharing passengers to encourage commuters participating in ridesharing. The values of compensations are always larger than 0.1. As a result, the platform makes zero profit and the system's disutility is \$12,824, which is slightly larger than that when the platform minimizes the system disutility.

5.2. Ridesharing participants commute with no queue

When ridesharing participants commute with no queue, if the platform aims to maximize the profit, we obtain a solution with $N_1 = 1435$ and $N_2 = 283$, which means there are 565 commuters participating in ridesharing. The departure rate from the origin at time t is shown in Fig. 6(a), where the overall congestion time window is [5.8, 8.6], indicating the duration is 2.8 h. The corresponding travel time patterns are shown in Fig. 6(b), where we find the queuing time is zero for ridesharing drivers and positive for solo drivers, indicating that the queue is only experienced by solo drivers. The charging and

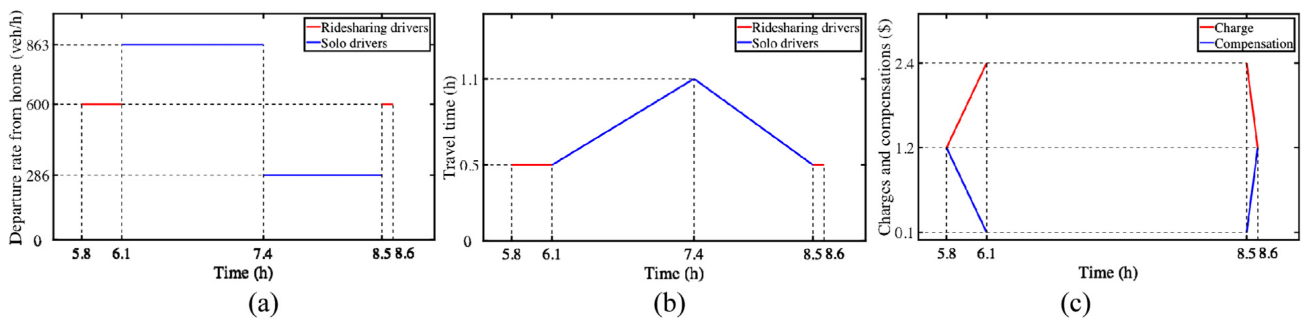

Fig. 6. Platform maximizes the profit when ridesharing participants commute with no queue.

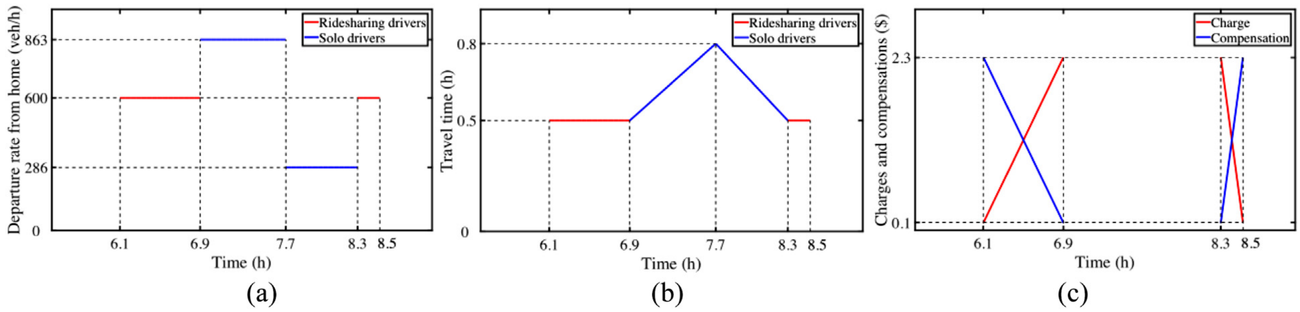

Fig. 7. Platform minimizes the system disutility with zero profit when ridesharing participants commute with no queue.

compensation curves are given in Fig. 6(c). Note that the values of charging are equal to that of compensation only at the earliest and latest departure times; beyond those times, the charges are always larger than the compensations, which are consistent with our analyses in Section 4.1. The resulting profit of the platform is $318 and the system disutility is $21,101.

If the platform aims to minimize the system disutility with zero profit, we obtain $N_1 = 869$ and $N_2 = 565$. That is to say, there are 1131 commuters participating in ridesharing. The departure rate from the origin at time t is shown in Fig. 7(a), where we find the overall congestion time window is [6.1, 8.5] with the duration 1.6 h. Compared to that when the platform maximizes the profit, the larger number of vehicular traffic leads to longer duration of departure time window. For ridesharing participants, the departure rate equals to the capacity during [6.1, 6.9] for early arrivals and [8.3, 8.5] for late arrivals. While for solo drivers, their departure time windows are [6.9, 7.7] for early arrivals and [7.7, 8.3] for late arrivals. Note the departure rates in both Figs. 6(a) and 7(a) share similar patterns, and the main difference between them is the duration. This verifies our earlier analyses in Remark 3. The travel time patterns are shown in Fig. 7(b), and the corresponding charging and compensation curves are portrayed in Fig. 7(c). It is observed that the charges are increasing for early arrivals and decreasing for late arrivals, while the compensations paid to the ridesharing drivers behave in an opposite way. The system disutility is $16,919.

5.3. Sensitivity analysis

Sensitivity analysis is conducted here by varying the input parameters of inconvenience cost (h_r for ridesharing drivers and h_p for passengers), and the induced impacts on system disutility and platform's profit are provided. The durations of departure time window separately for ridesharing drivers and solo drivers are analyzed when ridesharing participants commute with no queue.

Let the sum of the inconvenience cost parameters for drivers and passengers vary from zero to the fuel loss parameter, the induced changes of the system disutility under different objectives of the platform are shown in Fig. 8. It reveals that for all cases, the larger the inconvenience cost parameter, the greater the system disutility; while the nonlinear curve suggests that the system disutility increases more dramatically when the platform minimizes that with zero profit. When the platform maximizes the profit, the contour of the profit with the changes of the inconvenience cost parameters is plotted in Fig. 9, where we find larger inconvenience cost parameter lead to smaller platform's profit.

When ridesharing participants commute with no queue, in all cases shown in Fig. 10, the duration of departure time window for solo drivers increases with the change of the sum of inconvenience cost parameters, while the duration of departure time window for ridesharing drivers behaves oppositely. This implies that the number of ridesharing drivers decreases with the change of the sum of inconvenience cost parameters. With Fig. 10, under the given inconvenience cost parameters, the

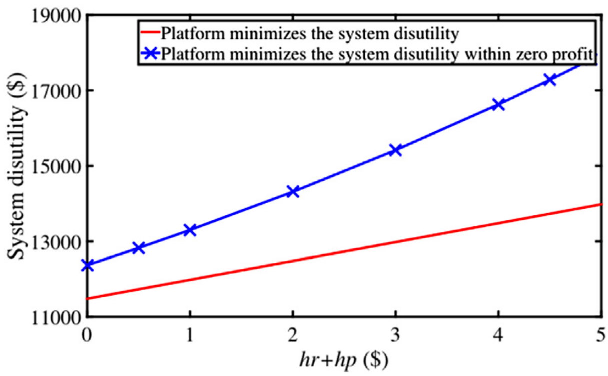

Fig. 8. System disutility with VCS.

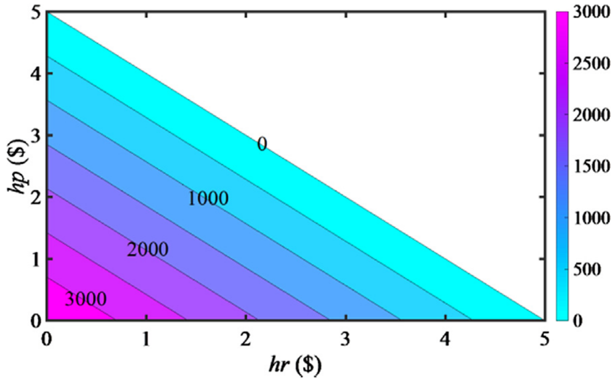

Fig. 9. Contours of platform's profit when the platform maximizes the profit.

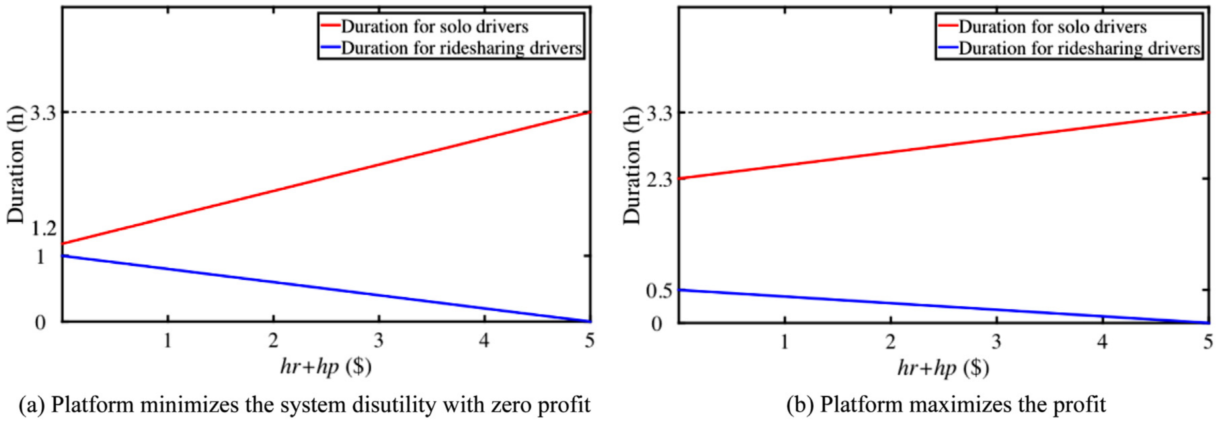

Fig. 10. The durations of departure time window for ridesharig drivers and solo drivers when ridesharing participants commute with no queue.

larger duration of departure time window for solo drivers when the platform maximizes profit compared to that when the platform minimizes the system disutility with zero profit, suggesting that the number of ridesharing participants is smaller if the platform maximizes its profit. Moreover, we find that the duration of the whole study period when the platform minimizes the system disutility with zero profit is shorter than that when the platform maximizes its profit. This verifies our earlier analyses in Remark 3.

6. Conclusions

Charging and compensation scheme is one of the most important components in ridesharing. To the best of the authors' knowledge, this paper represents the first effort to develop the variable-ratio charging-compensation scheme (VCS) for dynamic ridesharing. Compared to the fixed-ratio charging-compensation scheme (FCS) in existing studies, VCS allows more flexibility for the ridesharing platform to reach its desirable objectives, such as to minimize the system disutility or maximize the platform's profit, without imposing additional transportation management strategies such as road pricing. This feature (i.e., no pricing scheme is needed) may make it easier to implement VCS in practice, by considering the difficulty of deploying road pricing in many cities around the world. For given charging-compensation scheme, we first formulated the morning commute DUE problem on a single bottleneck as a DCS. To derive the optimal VCS, we developed a bi-level problem, with the lower level being the DCS-based DUE problem and the upper level to optimize for the VCS to achieve the platform's desirable objectives, which include (1) minimizing the system disutility; (2) maximizing the platform's profit; and (3) minimizing the system disutility with zero profit of the platform. Since the analyses are conducted on a single bottleneck, the problem can be solved on a single level. We found that the system optimum can be obtained when the platform minimize the system disutility by designing certain VCS, and the optimal VCS uniquely exists when the platform maximize its profit. To find the optimal VCS when the platform minimizes the system disutility with a zero profit, the problem can be solved as a nonlinear optimization problem. In this way, we provide a new perspective for the ridesharing platform to manage the dynamic ridesharing system and achieve the desirable objectives with VCS.

A special case, when ridesharing participants commute with no queue, was also studied. We found that in this case, the ridesharing scheme should be practiced at the two tails of the departure time window and the congestion is only experienced by solo drivers departing in the middle of the window. To find the optimal VCS, we examined the problem analytically and found that there should be smaller number of commuters involved in ridesharing when the platform maximizes its profit, leading to longer duration of departure time window for ridesharing drivers and longer duration of the whole study period, compared to those when the platform minimizes the system disutility with zero profit. Furthermore, the maximum profit is independent of the travel demand of the corridor. Findings of this research are expected to shed useful insights on how to design VCS and how to operate/manage a ridesharing platform under different objectives. The results also show that VCS can help the platform achieve those objectives without the use of pricing schemes.

There are several limitations in this study. First all commuters are assumed to have their own cars and can choose to participate in ridesharing or not. This assumption needs to be relaxed to recognize the fact that some commuters do not own cars. For this, ridesharing with elastic demand can be studied in future research. Second, for simplicity, a single ridesharing platform is assumed in our model. This assumption is idealized especially in a competitive market. For this, economic analyses concerning monopolistic competition are necessary with two or more ridesharing platforms introduced into the ridesharing market. Other future research directions can focus on relaxing the assumptions (A1)–(A5) in terms of homogeneous commuters, fixed ridership, and extensions to more general (multimodal) transportation networks. We will investigate these extensions and results may be reported in subsequent papers.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.trb.2019.03.006.

Appendix A. Computing results of the platform's profit in Eq. (22)

$$\begin{aligned}\pi &= \int_{t\in T} ((f-h_r-h_p)\tau(t))u(t)\mathrm{d}t.\\ &= (f-h_r-h_p)\left(\begin{aligned}&\int_{t_1}^{t_2}\left(\frac{\beta}{\alpha-\beta+f}(t-t_1)+\tau_0\right)\frac{\alpha+f}{\alpha-\beta+f}s\mathrm{d}t\\ &+\int_{t_2}^{t_3}\left(\frac{-\gamma}{\alpha+\gamma+f}(t-t_3)+\tau_0\right)\frac{\alpha+f}{\alpha+\gamma+f}s\mathrm{d}t\end{aligned}\right)\end{aligned}$$

Substituting t_1, t_2, t_3 in Eq. (20) into above, we get

$$\int_{t_1}^{t_2}\left(\frac{\beta}{\alpha-\beta+f}(t-t_1)+\tau_0\right)\frac{\alpha+f}{\alpha-\beta+f}s\mathrm{d}t = \frac{\gamma}{\beta+\gamma}\frac{N}{2}\tau_0 + \frac{\beta\gamma^2}{(\alpha+f)(\beta+\gamma)^2}\frac{N^2}{8s}$$

and

$$\int_{t_2}^{t_3}\left(\frac{-\gamma}{\alpha+\gamma+f}(t-t_3)+\tau_0\right)\frac{\alpha+f}{\alpha+\gamma+f}s\mathrm{d}t = \frac{\beta}{\beta+\gamma}\frac{N}{2}\tau_0 + \frac{\gamma\beta^2}{(\alpha+f)(\beta+\gamma)^2}\frac{N^2}{8s}$$

Then we conclude that the platform's profit is

$$\pi = (f-h_r-h_p)\left(\frac{N}{2}\tau_0 + \frac{\beta\gamma}{(\alpha+f)(\beta+\gamma)}\frac{N^2}{8s}\right).$$

Appendix B. Computing results of the platform's profit in Eq. (46)

$$\begin{aligned}\pi &= \int_{t_1}^{t_3} (m_p(t)-m_r(t))u(t)\mathrm{d}t\\ &= s\int_{t_1}^{t_4}(-2\beta(t^*-t-\tau_0))\mathrm{d}t + s\int_{t_5}^{t_3}(-2\gamma(t+\tau_0-t^*))\mathrm{d}t\\ &\quad+\left(\frac{2\beta\gamma}{\beta+\gamma}\frac{N}{s}-(f-h_r-h_p)\tau_0\right)(t_4-t_1+t_3-t_5).\end{aligned} \tag{58}$$

The first part of Eq. (58) is computed as

$$s\int_{t_1}^{t_4}(-2\beta(t^*-t-\tau_0))\mathrm{d}t+s\int_{t_5}^{t_3}(-2\gamma(t+\tau_0-t^*))\mathrm{d}t$$
$$=s\Big(-2\beta(t^*-\tau_0)(t_4-t_1)+\beta\big((t_4)^2-(t_1)^2\big)+2\gamma(t^*-\tau_0)(t_3-t_5)-\gamma\big((t_3)^2-(t_5)^2\big)\Big)$$
$$=\frac{\tau_0}{2}(f-h_r-h_p)\left(\frac{3\tau_0 s}{2}(f-h_r-h_p)\frac{\beta+\gamma}{\beta\gamma}-2N\right). \tag{59}$$

While the second part of Eq. (58) is

$$\left(\frac{2\beta\gamma}{\beta+\gamma}\frac{N}{s}-(f-h_r-h_p)\tau_0\right)(t_4-t_1+t_3-t_5)$$
$$=\left(\frac{2\beta\gamma}{\beta+\gamma}N-(f-h_r-h_p)\tau_0 s\right)\cdot\frac{\beta+\gamma}{2\beta\gamma}(f-h_r-h_p)\tau_0$$
$$=N(f-h_r-h_p)\tau_0-\frac{\beta+\gamma}{2\beta\gamma}((f-h_r-h_p)\tau_0)^2 s. \tag{60}$$

With Eqs. (59) and (60), we get the platform's profit as follows:

$$\pi=\frac{1}{4}\frac{\beta+\gamma}{\beta\gamma}((f-h_r-h_p)\tau_0)^2 s.$$

Appendix C. Computing results of the platform's profit in Eq. (48)

$$\pi=\int_{t_1}^{t_3}(m_p(t)-m_r(t))u(t)\mathrm{d}t$$
$$=s\int_{t_1}^{t_4}(-2\beta(t^*-t-\tau_0))\mathrm{d}t+s\int_{t_5}^{t_3}(-2\gamma(t+\tau_0-t^*))\mathrm{d}t$$
$$+\left((f-h_r-h_p)\tau_0 s+\frac{2\beta\gamma}{\beta+\gamma}N_s\right)(t_4-t_1+t_3-t_5). \tag{61}$$

Together with (36) and (38), the first part in Eq. (61) is

$$s\int_{t_1}^{t_4}(-2\beta(t^*-t-\tau_0))\mathrm{d}t+s\int_{t_5}^{t_3}(-2\gamma(t+\tau_0-t^*))\mathrm{d}t$$
$$=s\Big(-2\beta(t^*-\tau_0)(t_4-t_1)+\beta\big((t_4)^2-(t_1)^2\big)+2\gamma(t^*-\tau_0)(t_3-t_5)-\gamma\big((t_3)^2-(t_5)^2\big)\Big)$$
$$=s\left(\frac{\beta\gamma}{\beta+\gamma}\left(\frac{N_s}{s}\right)^2-(\beta+\gamma)(t^*-\tau_0-t_1)^2-\gamma\frac{(N+N_s)^2}{4s^2}+\frac{\gamma}{s}(t^*-\tau_0-t_1)(N+N_s)\right).$$

The second part of Eq. (61) is

$$\left((f-h_r-h_p)\tau_0 s+\frac{2\beta\gamma}{\beta+\gamma}N_s\right)(t_4-t_1+t_3-t_5)=\left((f-h_r-h_p)\tau_0 s+\frac{2\beta\gamma}{\beta+\gamma}N_s\right)\frac{N-N_s}{2s}.$$

Above all, the profit of the platform computed as functions of t_1 and N_s as below,

$$\pi=-(\beta+\gamma)(t^*-\tau_0-t_1)^2 s+\gamma(N+N_s)(t^*-\tau_0-t_1)$$
$$-\gamma\frac{(N+N_s)^2}{4s}+\frac{\beta\gamma}{\beta+\gamma}\frac{NN_s}{s}+\frac{N-N_s}{2}(f-h_r-h_p)\tau_0.$$

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