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A HYBRID FUZZY APPROACH TO FUZZY MULTI-ATTRIBUTE GROUP DECISION-MAKING

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The paper investigates fuzzy multi-attribute group decision-making (FMAGDM) problems. The important weights of the attributes and the ratings of the alternatives with respect to each attribute provided by multiple decision-makers are described by the linguistic variables expressed in triangular fuzzy numbers or trapezoidal fuzzy numbers. A hybrid fuzzy approach is proposed, which assesses each alternative in terms of distance measure calculated by a modified VIKOR method as well as similarity measure calculated by a modified gray relational analysis (GRA) method, to the positive ideal alternative and the negative ideal alternative. A new relative closeness coefficient is established to rank alternatives by aggregating the distance and the similarity measures. Two numerical examples for reverse logistics applications are presented to illustrate the proposed method.

Keywords: Fuzzy multi-attribute group decision-making (FMAGDM); linguistic variables; modified VIKOR; modified gray relational analysis (GRA); relative closeness coefficient; reverse logistics.

1. Introduction

Multi-attribute decision-making or multi-criteria decision-making (MADM/MCDM) problems select an appropriate option from a set of feasible alternatives based on the features of all evaluation attributes.[1] In classical MADM methods, such as TOPSIS by Hwang and Yoon,[1] SAW by MacCrimon,[2] and VIKOR by Opricovic,[11,12] the importance weights of attributes and the ratings of alternatives with respect to each attribute are assumed to be known. However, it is difficult to rank the alternatives with crisp values since human judgments including preferences are often vague.[37,38] A more realistic approach may be to use linguistic assessments instead of crisp values, that is, the alternatives are assessed by means of linguistic variables.[3–6,37,38] Moreover, multiple decision-makers (DMs) rather than a single DM are often involved in many decision-making processes.[7–10] The presence of multiple criteria and multiple DMs will increase the complexity of such processes. In this research, we consider these types of fuzzy multi-attribute group decision-making (FMAGDM) problems.

VIKOR is a widely used MADM method developed by Opricovic.[11,12] It focuses on ranking and selecting from a set of alternatives, and determines compromise solution to a problem by introducing the multi-attribute ranking index based on the particular measure of "closeness" to the "ideal" solution with conflicting attributes,[15,16] so as to help the DMs to reach a final solution. The multi-attribute measure for compromise ranking is developed from the *Lp-metric* that is used as an aggregating function in compromise programming.[13,14] Opricovic and Tzeng[15–17] compared the VIKOR method with several traditional MADM methods (TOPSIS, ELECTRE, PROMETHEE, and DEA) and pointed out its advantages. In recent years, VIKOR method has been successfully adopted to solve various MADM problems and demonstrated satisfactory results.[18–24]

Meanwhile, several researchers have applied VIKOR method to solve FMAGDM problems with linguistic variables.[25–30] Some have defuzzified fuzzy ratings and weights into crisp values by the center of gravity method and then employed classical VIKOR method to evaluate alternatives,[28–30] whereby the defuzzification will lose some fuzzy information since it avoids some operational laws of fuzzy sets. To reduce the loss of fuzzy information, others have used modified VIKOR methods with fuzzy data to evaluate alternatives.[25–27] In Buyukozkan's stuides,[25,26] linguistic variables were expressed in triangular fuzzy numbers. The final evaluation value was calculated as triangular fuzzy number and ranked by the area compensation method. In Chen and Wang's works,[27] linguistic variables were also expressed by triangular fuzzy numbers. The final evaluation value was calculated as triangular fuzzy number and ranked by the method of the maximizing set and minimizing set.

However, some problems are encountered in their researches. First, they combined fuzzy set theory to extend the classical VIKOR method in fuzzy environments. This involved considerable computations in the processes since there are enormous positive or negative fuzzy number operations as discussed in Ref. 31. Second, final evaluation values were ranked by fuzzy ranking methods. There are many fuzzy ranking methods presented in Ref. 32 and different methods employed may generate different and inconsistent results. Finally, these modified VIKOR methods only consider the shortest distance of each alternative from the positive ideal alternative.

Furthermore, the distance measure of each alternative from the positive ideal alternative only expresses the location relationship between each alternative and the positive ideal alternative, but does not reflect the trend. Gray relational analysis (GRA), proposed by Deng,[33] can be used to capture the trend correlations between the referential sequence (alternative) and other compared alternatives of a system.[34] Olson and Wu[35] and Wu[36] applied the GRA method to evaluate the similarity measure of each alternative to the positive ideal alternative in fuzzy environment.

In this paper, we propose a hybrid fuzzy approach to solve FMAGDM problems with linguistic variables. The proposed approach establishes a new relative closeness coefficient by combining distance measure with similarity measure to rank alternatives. The new relative closeness coefficient reflects the fact that the most satisfying alternative should simultaneously have the shortest distance from the positive ideal alternative and the largest distance from the negative ideal alternative. Moreover, it also reflects that the best alternative should have the largest similarity to the positive ideal alternative and the least similarity to the negative ideal alternative, simultaneously. Based on the distance of two fuzzy numbers calculated by the vertex method outlined in Refs. 37 and 38, a modified VIKOR method and a modified GRA method are presented to calculate the distance and the similarity of each alternative to the positive ideal alternative and negative ideal alternative, respectively. Finally, two examples in reverse logistics are presented to demonstrate the applications of the proposed method.

The rest of this paper is structured as follows. In Sec. 2, we present a brief introduction to linguistic variables and fuzzy set theory. Section 3 presents the proposed hybrid fuzzy approach. Several numerical example problems in the area of reverse logistics system design solved by the proposed hybrid approach are shown in Sec. 4. Conclusions and future research topics are given in the final section.

2. Linguistic Variables and Fuzzy Set Theory

In this section, a brief introduction to linguistic variables and fuzzy set theory is presented and briefly explained.

2.1. *Linguistic variable*

A linguistic variable is a variable whose values are expressed in linguistic terms.[39,42] Linguistic variables are useful when decision problems are complex or difficult to describe properly using conventional quantitative expressions. For example, the performance ratings of alternatives on qualitative attributes could be described by linguistic variable as very poor, poor, medium poor, fair, medium good, good, very good, etc. Such linguistic values can be represented using positive triangular fuzzy numbers or trapezoid fuzzy numbers. The value of a linguistic variable can be quantified and treated by mathematical operations using fuzzy set theory.

2.2. *Fuzzy set theory*

Some definitions in fuzzy set theory are reviewed as follows (see Refs. 37–42 for more details):

Definition 2.1. A positive triangular fuzzy number $\tilde{M}$ can be defined as (m_1, m_2, m_3), where the membership function $\mu_{\tilde{M}}(x)$ of $\tilde{M}$ is given by

$$\mu_{\tilde{M}}(x) = \begin{cases} 0, & x < m_1 \\ (x - m_1)/(m_2 - m_1), & m_1 \le x \le m_2 \\ (m_3 - x)/(m_3 - m_2), & m_2 \le x \le m_3 \\ 0, & x > m_3. \end{cases} \tag{2.1}$$

Let $\tilde{M} = (m_1, m_2, m_3)$ and $\tilde{N} = (n_1, n_2, n_3)$ be two positive triangular fuzzy numbers, and $r > 0$ be a positive real number; some main operations of $\tilde{M}$ and $\tilde{N}$ can be expressed as follows:

(1) $\tilde{M}(+)\tilde{N} = [m_1 + n_1, m_2 + n_2, m_3 + n_3]$;
(2) $\tilde{M}(-)\tilde{N} = [m_1 - n_3, m_2 - n_2, m_3 - n_1]$;
(3) $\tilde{M}(\times)\tilde{N} \cong [m_1 n_1, m_2 n_2, m_3 n_3]$;
(4) $\tilde{M}(\div)\tilde{N} \cong [m_1/n_3, m_2/n_2, m_3/n_1]$;
(5) $r(\times)\tilde{M} \cong [rm_1, rm_2, rm_3]$;
(6) $(\tilde{M})^{-1} \cong [1/m_3, 1/m_2, 1/m_1]$.

Definition 2.2. A positive trapezoidal fuzzy number $\tilde{M}$ can be defined as (m_1, m_2, m_3, m_4), where the membership function $\mu_{\tilde{M}}(x)$ of $\tilde{M}$ is given by

$$\mu_{\tilde{M}}(x) = \begin{cases} 0, & x < m_1 \\ (x - m_1)/(m_2 - m_1), & m_1 \leq x \leq m_2 \\ 1, & m_2 \leq x \leq m_3 \\ (m_4 - x)/(m_4 - m_3) & m_3 \leq x \leq m_4 \\ 0, & x > m_4. \end{cases} \tag{2.2}$$

Let $\tilde{M} = (m_1, m_2, m_3, m_4)$ and $\tilde{N} = (n_1, n_2, n_3, n_4)$ be two positive trapezoidal fuzzy numbers, and $r > 0$ be a positive real number; some main operations of $\tilde{M}$ and $\tilde{N}$ can be expressed as follows:

(1) $\tilde{M}(+)\tilde{N} = [m_1 + n_1, m_2 + n_2, m_3 + n_3, m_4 + n_4]$;
(2) $\tilde{M}(-)\tilde{N} = [m_1 - n_4, m_2 - n_3, m_3 - n_2, m_4 - n_1]$;
(3) $\tilde{M}(\times)\tilde{N} \cong [m_1 n_1, m_2 n_2, m_3 n_3, m_4 n_4]$;
(4) $\tilde{M}(\div)\tilde{N} \cong [m_1/n_4, m_2/n_3, m_3/n_2, m_4/n_1]$;
(5) $r(\times)\tilde{M} \cong [rm_1, rm_2, rm_3, rm_4]$;
(6) $(\tilde{M})^{-1} \cong [1/m_4, 1/m_3, 1/m_2, 1/m_1]$.

Definition 2.3. Let $\tilde{M} = (m_1, m_2, m_3)$ and $\tilde{N} = (n_1, n_2, n_3)$ be two triangular fuzzy numbers. Then the distance between them can be calculated by using the vertex method as

$$d(\tilde{M}, \tilde{N}) = \sqrt{\frac{1}{3}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2]}. \tag{2.3}$$

Let $\tilde{M} = (m_1, m_2, m_3, m_4)$ and $\tilde{N} = (n_1, n_2, n_3, n_4)$ be two triangular fuzzy numbers. Then the distance between them can be calculated by using the vertex method as

$$d(\tilde{M}, \tilde{N}) = \sqrt{\frac{1}{4}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 + (m_4 - n_4)^2]}. \tag{2.4}$$

3. The Proposed Hybrid Fuzzy Approach

We propose a hybrid fuzzy approach to solve FMAGDM problems by incorporating a modified VIKOR method and a modified GRA method. The importance weights of attributes and the ratings of alternatives with respect to each attribute are described by linguistic variables, which can be expressed by triangular fuzzy numbers or trapezoidal fuzzy numbers. More specifically, the considered FMAGDM problem can be described by the following:

(1) A set of K DMs (experts), $D = \{D_1, D_2, \ldots, D_K\}$;
(2) A set of m possible alternatives, $A = \{A_1, A_2, \ldots, A_m\}$;
(3) A set of n attributes, $C = \{C_1, C_2, \ldots, C_n\}$ to measure the performances of the alternatives;
(4) The performances rating of A_i with respect to C_j provided by D_k is expressed as $\tilde{X}^k = \{\tilde{x}_{ij}^k\}$, $i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, K$;
(5) The importance weight of attribute C_j provided by D_k is expressed as $\tilde{W}^k = \{\tilde{w}_j^k\}$, $j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, K$.

The detailed steps of the proposed approach are given below.

Step 1. Identify the objectives of the decision-making process, arrange the decision-making group, and define a finite set of relevant attributes.

Step 2. Identify the appropriate linguistic variables.

In this step, we must define the appropriate linguistic variables for the importance weights of attributes and the fuzzy ratings for alternatives with regard to each attribute. These linguistics variables can be expressed in positive triangular fuzzy numbers or in positive trapezoidal fuzzy numbers.

Step 3. Determine the aggregate fuzzy weight $\tilde{w}_j$ of attribute $C_j,\ j = 1, 2, \ldots, n$, and the aggregate fuzzy rating $\tilde{x}_{ij}$ of alternative $A_i,\ i = 1, 2, \ldots, m$, with respect to attribute C_j.

(1) *Using triangular fuzzy numbers*: It is assumed that $\tilde{x}_{ij}^k = (x_{ij1}^k, x_{ij2}^k, x_{ij3}^k)$ and $\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k)$, respectively. Then, the aggregate fuzzy rating $\tilde{x}_{ij}$ of alternatives and the aggregate fuzzy weight $\tilde{w}_j$ are computed by

$$\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}),$$

where

$$x_{ij1} = \frac{\sum_{k=1}^{K} x_{ij1}^k}{K}, \quad x_{ij2} = \frac{\sum_{k=1}^{K} x_{ij2}^k}{K}, \quad x_{ij3} = \frac{\sum_{k=1}^{K} x_{ij3}^k}{K} \tag{3.1}$$

and

$$\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}),$$

where

$$w_{j1} = \frac{\sum_{k=1}^{K} w_{j1}^k}{K}, \quad w_{j2} = \frac{\sum_{k=1}^{K} w_{j2}^k}{K}, \quad w_{j3} = \frac{\sum_{k=1}^{K} w_{j3}^k}{K}. \tag{3.2}$$

(2) *Using trapezoidal fuzzy numbers*: Assume that $\tilde{x}_{ij}^k = (x_{ij1}^k, x_{ij2}^k, x_{ij3}^k, x_{ij4}^k)$ and $\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k)$, respectively. The aggregate fuzzy rating $\tilde{x}_{ij}$ of alternatives and the aggregate fuzzy weight $\tilde{w}_j$ are then computed by

$$\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}),$$

where

$$\begin{aligned} x_{ij1} &= \min_k \left\{x_{ij1}^k\right\}, \quad x_{ij2} = \frac{\sum_{k=1}^{K} x_{ij2}^k}{K}, \\ x_{ij3} &= \frac{\sum_{k=1}^{K} x_{ij3}^k}{K}, \quad x_{ij4} = \max_k \left\{x_{ij4}^k\right\}, \end{aligned} \tag{3.3}$$

and

$$\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4}),$$

where

$$\begin{aligned} w_{j1} &= \min_k \left\{w_{j1}^k\right\}, \quad w_{j2} = \frac{\sum_{k=1}^{K} w_{j2}^k}{K}, \\ w_{j3} &= \frac{\sum_{k=1}^{K} w_{j3}^k}{K}, \quad w_{j4} = \max_k \left\{w_{j4}^k\right\}. \end{aligned} \tag{3.4}$$

The FMAGDM problem can then be expressed in the matrix form as

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \quad \tilde{W} = [\tilde{w}_1 \quad \tilde{w}_2 \quad \cdots \quad \tilde{w}_n]. \tag{3.5}$$

Step 4. Construct the normalized fuzzy decision matrix $\tilde{F} = (\tilde{f}_{ij})_{m \times n}$.

Scaling is required if a fuzzy number expressing linguistic variables is greater than 1.0; the linear scale transformation is used here to transform the various attributes into comparable scales, which does not depend on the evaluation unit of an attribute function.[15,18]

(1) *Using triangular fuzzy numbers*: $j \in B$:

$$\tilde{f}_{ij} = \left(\frac{x_{ij1} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \frac{x_{ij2} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \frac{x_{ij3} - x_j^{\min}}{x_j^{\max} - x_j^{\min}} \right) = (f_{ij1}, f_{ij2}, f_{ij3}), \tag{3.6}$$

$j \in C$:

$$\tilde{f}_{ij} = \left(\frac{x_j^{\max} - x_{ij3}}{x_j^{\max} - x_j^{\min}}, \frac{x_j^{\max} - x_{ij2}}{x_j^{\max} - x_j^{\min}}, \frac{x_j^{\max} - x_{ij1}}{x_j^{\max} - x_j^{\min}} \right) = (f_{ij1}, f_{ij2}, f_{ij3}), \tag{3.7}$$

where $x_j^{\max} = \max_i x_{ij3},\ x_j^{\min} = \min_i x_{ij1},\ i = 1, 2, \ldots, m$; B and C are the sets of benefit attributes and cost attributes, respectively.

(2) *Using trapezoidal fuzzy numbers*: $j \in B$:

$$\begin{aligned} \tilde{f}_{ij} &= \left(\frac{x_{ij1} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \frac{x_{ij2} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \frac{x_{ij3} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \frac{x_{ij4} - x_j^{\min}}{x_j^{\max} - x_j^{\min}} \right) \\ &= (f_{ij1}, f_{ij2}, f_{ij3}, f_{ij4}), \end{aligned} \tag{3.8}$$

$j \in C$:

$$\begin{aligned} \tilde{f}_{ij} &= \left(\frac{x_j^{\max} - x_{ij4}}{x_j^{\max} - x_j^{\min}}, \frac{x_j^{\max} - x_{ij3}}{x_j^{\max} - x_j^{\min}}, \frac{x_j^{\max} - x_{ij2}}{x_j^{\max} - x_j^{\min}}, \frac{x_j^{\max} - x_{ij1}}{x_j^{\max} - x_j^{\min}} \right) \\ &= (f_{ij1}, f_{ij2}, f_{ij3}, f_{ij4}), \end{aligned} \tag{3.9}$$

where $x_j^{\max} = \max_i x_{ij4},\ x_j^{\min} = \min_i x_{ij1},\ i = 1, 2, \ldots, m$; B and C are the sets of benefit attributes and cost attributes, respectively.

Step 5. Construct the weighted normalized fuzzy decision matrix $Z = (\tilde{z}_{ij})_{m \times n}$.

For triangular fuzzy numbers,

$$\tilde{z}_{ij} = \tilde{w}_j(\times)\tilde{f}_{ij} = (w_{j1} f_{ij1}, w_{j2} f_{ij2}, w_{j3} f_{ij3}) = (z_{ij1}, z_{ij2}, z_{ij3}) \tag{3.10}$$

And for trapezoidal fuzzy numbers,

$$\tilde{z}_{ij} = \tilde{w}_j(\times)\tilde{f}_{ij} = (w_{j1} f_{ij1}, w_{j2} f_{ij2}, w_{j3} f_{ij3}, w_{j4} f_{ij4}) = (z_{ij1}, z_{ij2}, z_{ij3}, z_{ij4}). \tag{3.11}$$

Step 6. Determine the fuzzy positive-ideal alternative (FPIA, A^+) and the negative-ideal alternative (FNIA, A^-), which can be defined as

$$A^+ = (\tilde{z}_1^+, \tilde{z}_2^+, \ldots, \tilde{z}_n^+); \quad A^- = (\tilde{z}_1^-, \tilde{z}_2^-, \ldots, \tilde{z}_n^-), \tag{3.12}$$

where

$$\tilde{z}_j^+ = (z_{j1}^+, z_{j2}^+, z_{j3}^+), \quad z_{j1}^+ = z_{j2}^+ = z_{j3}^+ = \max_i \{z_{ij3}\};$$

$$\tilde{z}_j^- = (z_{j1}^-, z_{j2}^-, z_{j3}^-), \quad z_{j1}^- = z_{j2}^- = z_{j3}^- = \min_i \{z_{ij1}\} \text{ for triangular fuzzy numbers;}$$

and

$$\tilde{z}_j^+ = (z_{j1}^+, z_{j2}^+, z_{j3}^+, z_{j4}^+), \quad z_{j1}^+ = z_{j2}^+ = z_{j3}^+ = z_{j4}^+ = \max_i \{z_{ij4}\};$$

$$\tilde{z}_j^- = (z_{j1}^-, z_{j2}^-, z_{j3}^-, z_{j4}^-), \quad z_{j1}^- = z_{j2}^- = z_{j3}^- = z_{j4}^- = \min_i \{z_{ij1}\}$$

for trapezoidal fuzzy numbers.

Step 7. Calculate the distance measure of each alternative to the FPIA and FNIA by the modified VIKOR method.

Calculate the values of S_i^+, S_i^-, R_i^+, R_i^- for each alternative by the relations:

$$S_i^+ = \sum_{j=1}^{n} d(\tilde{z}_{ij}, \tilde{z}_j^+), \quad R_i^+ = \max_j[d(\tilde{z}_{ij}, \tilde{z}_j^+)], \tag{3.13}$$

$$S_i^- = \sum_{j=1}^{n} d(\tilde{z}_{ij}, \tilde{z}_j^-), \quad R_i^- = \max_j[d(\tilde{z}_{ij}, \tilde{z}_j^-)], \tag{3.14}$$

where S_i^+ and R_i^+ are used to formulate the ranking measurement of "group utility" and the "individual regret" between each alternative and FPIA, respectively; S_i^- and R_i^- are used to formulate the ranking measurement of "group utility" and the "individual regret" between each alternative and FNIA, respectively; $d(\cdot,\cdot)$ is the distance measurement between two fuzzy numbers given in Definition 3 in Sec. 2.2.

After S_i^+, S_i^-, R_i^+, and R_i^- are calculated, the distances Q_i^+ and Q_i^- of each alternative to the FPIA and FNIA can be determined by

$$Q_i^+ = v\frac{S_i^+ - \min_i S_i^+}{\max_i S_i^+ - \min_i S_i^+} + (1-v)\frac{R_i^+ - \min_i R_i^+}{\max_i R_i^+ - \min_i R_i^+}, \tag{3.15}$$

$$Q_i^- = v\frac{S_i^- - \min_i S_i^-}{\max_i S_i^- - \min_i S_i^-} + (1-v)\frac{R_i^- - \min_i R_i^-}{\max_i R_i^- - \min_i R_i^-}, \tag{3.16}$$

where $v(0 \leq v \leq 1)$ is introduced as the weight for the strategy of "maximum group utility" for the majority of attributes. When $v > 0.5$, the decision tends toward the maximum majority rule; and if $v < 0.5$, the decision tends toward the individual regret of the opponent. Here $v = 0.5$, the decision tends toward the consensus rule.

Step 8. Calculate the similarity measure of each alternative to the FPIA and FNIA by the modified GRA method.

Let the FPIA and FNIA be the referential sequences and each alternatives be the comparative sequence.

With respect to the jth attribute, the gray relational coefficient of each alternative to FPIA, expressed as $\xi(A_i(j),\ A^+(j))$, and the gray relational coefficient of each alternative to FNIA, expressed as $\xi(A_i(j),\ A^-(j))$, can be calculated as follows:

$$\xi(A_i(j), A^+(j)) = \frac{\min_i \min_j |\tilde{z}_{ij} - \tilde{z}_j^+| + \rho \max_i \max_j |\tilde{z}_{ij} - \tilde{z}_j^+|}{|\tilde{z}_{ij} - \tilde{z}_j^+| + \rho \max_i \max_j |\tilde{z}_{ij} - \tilde{z}_j^+|}, \tag{3.17}$$

$$\xi(A_i(j), A^-(j)) = \frac{\min_i \min_j |\tilde{z}_{ij} - \tilde{z}_j^-| + \rho \max_i \max_j |\tilde{z}_{ij} - \tilde{z}_j^-|}{|\tilde{z}_{ij} - \tilde{z}_j^-| + \rho \max_i \max_j |\tilde{z}_{ij} - \tilde{z}_j^-|}, \tag{3.18}$$

where $|\tilde{z}_{ij} - \tilde{z}_j^+| = d(\tilde{z}_{ij}, \tilde{z}_j^+)$ and $|\tilde{z}_{ij} - \tilde{z}_j^-| = d(\tilde{z}_{ij}, \tilde{z}_j^-)$; ρ is the distinguished coefficient ($\rho \in [0, 1]$), and always $\rho = 0.5$.

After $\xi(A_i(j),\ A^+(j))$ and $\xi(A_i(j),\ A^-(j))$ are calculated, the similarity measure of each alternative to FPIA and FNIA are computed by the following formulas:

$$r_i^+ = r(A_i,\ A^+) = \frac{1}{n}\sum_{j=1}^{n}\xi(A_i(j), A^+(j)), \tag{3.19}$$

$$r_i^- = r(A_i,\ A^-) = \frac{1}{n}\sum_{j=1}^{n}\xi(A_i(j), A^-(j)). \tag{3.20}$$

Step 9. Normalize the distance measure and the similarity measure of each alternative to FPIA and FNIA, and integrate them.

Because of the different scales of the distance measure and the similarity measure, they must be normalized. Some common normalization methods including vector normalization, linear normalization, and nonmonotonic normalization can be used.[1,43,44] The normalized values of M_i, M_{iN} can be derived by the linear normalization relation in our study:

$$M_{iN} = \frac{M_i}{\max\limits_i M_i}, \tag{3.21}$$

where M_{iN} stands for Q_{iN}^+, Q_{iN}^-, r_{iN}^+, and r_{iN}^-; M_i stands for Q_i^+, Q_i^-, r_i^+, and r_i^-.

It is known that the larger r_{iN}^+ or Q_{iN}^- is, the closer each alternative to the FPIA is; on the contrary, the larger r_{iN}^- or Q_{iN}^+ is, the farther each alternative to the FPIA is. We then calculate

$$D_i^+ = \alpha Q_{iN}^- + (1-\alpha) r_{iN}^+, \tag{3.22}$$

$$D_i^- = \alpha Q_{iN}^+ + (1-\alpha) r_{iN}^-, \tag{3.23}$$

where D_i^+ denotes the closeness degree of each alternative to the FPIA, D_i^- denotes the closeness degree of each alternative to the FNIA. α and $(1-\alpha)$ are the DM's preference to the distance measure and the similarity measure, respectively. Here, $0 \le \alpha \le 1$.

Step 10. Calculate the relative closeness coefficient of each alternative to FPIA, C_i^+, by the following formula:

$$C_i^+ = \frac{D_i^+}{(D_i^+ + D_i^-)}. \tag{3.24}$$

Step 11. Rank the set of feasible alternatives.

The set of feasible alternatives can be ranked by the descending order of the value of C_i^+.

4. Illustrative Examples

The example problems presented in this section are all in reverse logistics system applications. With increased public environmental concerns and stringent government regulations, reverse logistics research has received growing attention in recent years. There are several critical issues normally considered in reverse logistics system design, such as centralized return centers location selection, reverse manufacturing alternatives selection, etc. These problems are complex FMAGDM problems regarding time, quality, and quantity of returns. Therefore, we have chosen one example in each of these two reverse logistic subareas to illustrate the proposed hybrid approach.

4.1. *Example 1: Centralized return centers location selection*

Tuzkaya and Gülsün[45] proposed an integrated ANP-fuzzy TOPSIS method to select centralized return centers (CRCs) for processing used products in a reverse logistics network. The problem considers eight alternative CRCs and five attributes (Transportation (EC_1), Economical (EC_2), Environmental (EC_3), Social–political (EC_4), and Technical (EC_5)). A committee of five DMs (DM_1, DM_2, DM_3, DM_4, and DM_5) will make pairwise evaluation on the criteria for their relative impact, and also determine the ratings of eight alternative CRCs for the various attributes expressed in positive triangular fuzzy numbers. The method proposed in this paper can be used to solve this example problem.

The importance weight of the evaluation attribute for each DM is calculated by ANP method in Ref. 45. We use a transformation to obtain the aggregate fuzzy weight of the jth attribute by all K DMs as follows:

$$w_{j1} = \min_k\{w_j^k\}, \quad w_{j2} = \frac{\sum_{k=1}^{K} w_j^k}{K}, \quad w_{j3} = \max_k\{w_j^k\}, \qquad k = 1, 2, \ldots, K,$$

where w_j^k is the importance weight of the jth evaluation attribute by the kth DM.

We first transfer linguistic variables into triangular fuzzy numbers, and then calculate the aggregate fuzzy weight of attributes and fuzzy ratings of alternatives with respect to each attribute which are calculated according to Step 3 of the proposed approach, as summarized in Table 1.

The final evaluation values of C_i^+ (when $\rho = 0.5$, $v = 0.5$, and $\alpha = 0.5$) for all CRCs are calculated according to the computing steps from Steps 4 to 10, and are shown in Table 2.

We can obtain the ranking order of the CRCs from Table 2 based on the descending order of C_i^+ (Step 11) as $CRC_6 > CRC_3 > CRC_4 > CRC_7 > CRC_8 > CRC_2 > CRC_1 > CRC_5$. The ranking of CRCs is the same as the result in Ref. 45. It shows the validity of the proposed approach.

As shown in the procedure of the proposed approach, the values of α, v, and ρ can affect the final decision results. However, in our study, first, it is suitable to set the value of α to be 0.5 because the distance measure and the similarity

Table 1. Aggregate fuzzy weight of attributes and fuzzy ratings of alternatives.

	Attribute				
	EC_1	EC_2	EC_3	EC_4	EC_5
Weight	[0.35,0.44,0.53]	[0.01,0.05,0.09]	[0.14,0.18,0.24]	[0.06,0.12,0.18]	[0.10,0.21,0.34]
CRC_1	[0.2,0.8,2.2]	[0.4,1.8,3.8]	[0.4,1.8,3.8]	[0.0,0.4,1.8]	[5.4,7.4,8.8]
CRC_2	[4.2,6.2,8.2]	[5.0,7.0,8.6]	[5.8,7.8,9.4]	[6.2,8.2,9.6]	[4.6,6.6,8.4]
CRC_3	[7.4,9.0,9.8]	[7.8,9.4,10]	[7.0,8.6,9.4]	[4.2,6.2,8.0]	[5.8,7.8,9.4]
CRC_4	[7.4,9.0,9.8]	[8.6,9.8,10]	[7.8,9.4,10]	[3.8,5.8,7.6]	[5.0,7.0,8.6]
CRC_5	[0.0,0.4,1.8]	[0.4,1.8,3.8]	[0.0,0.6,2.2]	[2.0,3.2,4.8]	[3.2,5.0,6.8]
CRC_6	[6.2,8.2,9.6]	[5.4,7.4,9.0]	[6.2,8.2,9.6]	[8.2,9.6,10]	[7.4,9.0,9.8]
CRC_7	[5.4,7.4,8.8]	[5.0,7.0,8.8]	[4.2,6.2,8.0]	[6.2,8.2,8.8]	[7.8,9.2,9.8]
CRC_8	[4.6,6.6,8.2]	[4.6,6.6,8.4]	[4.2,6.2,8.2]	[5.4,7.4,8.8]	[5.4,7.2,8.6]

Table 2. The final evaluation values of the CRCs.

CRCs	FPIA		FNIA		D_i^+	D_i^-	C_i
	r_i^+	Q_i^+	r_i^-	Q_i^-			
CRC_1	0.6739	0.9223	0.8482	0.1764	0.5017	0.9466	0.3464
CRC_2	0.7783	0.2297	0.6466	0.7248	0.8437	0.4850	0.6350
CRC_3	0.8141	0.0179	0.6232	0.9857	0.9980	0.3657	0.7318
CRC_4	0.8138	0.0538	0.6295	0.9710	0.9903	0.3872	0.7189
CRC_5	0.6644	1.0000	0.8735	0	0.4064	1.0000	0.2890
CRC_6	0.8175	0	0.6121	0.9589	0.9864	0.3504	0.7379
CRC_7	0.7932	0.0991	0.6331	0.8425	0.9125	0.4120	0.6890
CRC_8	0.7713	0.2116	0.6568	0.7238	0.8389	0.4818	0.6352

measure are of the same importance. Second, the value of v is introduced as the weight for the strategy of "maximum group utility" for the majority of attributes, that is, it depends on the decision attitude (optimistic, pessimistic, or neutral) of the DM according to the features of the specific problems; therefore, the value of v is determined (in our study, we choose $v = 0.5$). Therefore, for the applications of the proposed method, the final decision results depend on the selecting value of ρ, which belongs to $[0, 1]$. In the following, we conduct a sensitivity analysis of ρ (where $\alpha = 0.5$ and $v = 0.5$) for Example 1. Table 3 shows the ranking of the CRCs

Table 3. The final evaluation values and the ranking of the crcs with different values of ρ.

CRC/ρ	0	0.1	0.3	0.5	0.7	0.9	1
CRC_1	0.2927(7)	0.3163(7)	0.3369(7)	0.3464(7)	0.3552(7)	0.3562(7)	0.3577(7)
CRC_2	0.7505(5)	0.6878(5)	0.6514(5)	0.6350(6)	0.6253(6)	0.6189(6)	0.6164(6)
CRC_3	0.8664(2)	0.7948(2)	0.7518(2)	0.7318(2)	0.7196(2)	0.7112(2)	0.7080(2)
CRC_4	0.8513(3)	0.7804(3)	0.7383(3)	0.7189(3)	0.7072(3)	0.6992(3)	0.6961(3)
CRC_5	0.2430(8)	0.2583(8)	0.2782(8)	0.2890(8)	0.2960(8)	0.3010(8)	0.3030(8)
CRC_6	0.8728(1)	0.8031(1)	0.7592(1)	0.7379(1)	0.7248(1)	0.7157(1)	0.7121(1)
CRC_7	0.8128(4)	0.7459(4)	0.7071(4)	0.6890(4)	0.6780(4)	0.6704(4)	0.6675(4)
CRC_8	0.7465(6)	0.6842(6)	0.6503(6)	0.6352(5)	0.6263(5)	0.6204(5)	0.6181(5)

with different values of ρ. In these cases, the best choice of CRC (CRC_6) does not change, although the ranking of the alternative CRCs changes slightly.

4.2. *Example 2: Reverse manufacturing alternatives selection*

Wadhwa *et al.*[46] proposed a fuzzy TOPSIS method to select a favorable reverse manufacturing alternative (RMA) for used-product return in an original equipment manufacturing (OEM) company. The problem considers five candidate alternatives (Remanufacturing (RMA_1), Recycling (RMA_2), Repair/Reuse (RMA_3), Cannibalization (RMA_4), and Refurbishing (RMA_5)) and five evaluation attribute (cost/time (C_1), environmental impact (C_2), market factor (C_3), quality factor (C_4), legislative impact (C_5)). A committee of four DMs (P_1, P_2, P_3, and P_n) is formed to determine the importance weights of the attributes and the ratings of the five candidate alternatives under the various attributes on the basis of the linguistic variables expressed in positive trapezoidal fuzzy numbers. The method proposed in this paper can be used to solve this example problem.

We first transfer linguistic variables into trapezoidal fuzzy numbers, and then calculate the aggregate fuzzy weights of attributes and fuzzy ratings of alternatives with respect to each attribute, which are calculated according to Step 3 of the proposed approach, as summarized in Table 4.

The final evaluation values of C_i^+ (when $\rho = 0.5$, $v = 0.5$, and $\alpha = 0.5$) for all reverse manufacturing alternatives are calculated according to the computing steps from Steps 4 to 10, and are shown in Table 5.

Table 4. Aggregate fuzzy weight of attributes and fuzzy ratings of alternatives.

	Attribute				
	C_1	C_2	C_3	C_4	C_5
Weight	[0.8,0.9,1,1]	[0.7,0.83,0.85,1]	[0.5,0.8,0.88,1]	[0.5,0.75,0.78,0.9]	[0.7,0.8,0.8,0.9]
RMA_1	[2,4.3,5,8]	[4,6.8,7,9]	[2,5.8,8.3,9]	[4,6.8,7,9]	[2,6.3,6.8,9]
RMA_2	[4,5.8,6.5,8]	[5,7.8,8.3,10]	[4,7.8,8.3,10]	[2,6.8,7,9]	[2,6.3,6.8,9]
RMA_3	[4,6.8,7,9]	[4,7.8,8.3,10]	[4,6.8,7,9]	[4,6.3,6.8,9]	[4,7,7.5,10]
RMA_4	[4,6.3,6.8,9]	[2,5.8,6.5,9]	[4,6.8,7,9]	[2,6.3,6.8,9]	[2,6.3,6.8,9]
RMA_5	[2,5.3,5.5,9]	[2,6.5,7.3,10]	[4,6.8,7,9]	[2,6.3,6.8,9]	[4,7,7.5,10]

Table 5. The final evaluation values of the reverse manufacturing alternatives (RMAs).

RMAs	FPIA		FNIA		D_i^+	D_i^-	C_i
	r_i^+	Q_i^+	r_i^-	Q_i^-			
RMA_1	0.8988	1.0000	0.8874	0	0.4795	0.9700	0.3308
RMA_2	0.8972	0.1197	0.8816	0.8742	0.9157	0.5267	0.6348
RMA_3	0.8728	0.0304	0.9110	1.0000	0.9656	0.4976	0.6599
RMA_4	0.9373	0.7169	0.9152	0.6623	0.8311	0.8431	0.4964
RMA_5	0.9341	0.6972	0.9441	0.4247	0.7107	0.8486	0.4558

Table 6. The final evaluation values and the ranking of the Rs with different values of ρ.

RMAs/ρ	0	0.1	0.3	0.5	0.7	0.9	1
RMA_1	0.3301(5)	0.3303(5)	0.3305(5)	0.3308(5)	0.3310(5)	0.3312(5)	0.3313(5)
RMA_2	0.6401(2)	0.6386(2)	0.6364(2)	0.6348(2)	0.6337(2)	0.6328(2)	0.6324(2)
RMA_3	0.6598(1)	0.6598(1)	0.6599(1)	0.6599(1)	0.6599(1)	0.6599(1)	0.6599(1)
RMA_4	0.4994(3)	0.4985(3)	0.4973(3)	0.4964(3)	0.4958(3)	0.4953(3)	0.4951(3)
RMA_5	0.4557(4)	0.4557(4)	0.4557(4)	0.4558(4)	0.4558(4)	0.4558(4)	0.4559(4)

We can obtain the ranking order of reverse manufacturing alternatives from Table 5 based on the descending order of the final evaluation value C_i^+ (Step 11) as $R_3 > R_2 > R_4 > R_5 > R_1$. The ranking order of reverse manufacturing alternatives is the same as the result in Ref. 46, showing also the validity of the proposed approach.

Below, we also conduct a sensitivity analysis of ρ (where $\alpha = 0.5$ and $v = 0.5$) for Example 2. Table 6 shows the ranking of the RMAs with different values of ρ. In these cases, the best choice of RMAs (RMA_3: Repair/Reuse) does not change, although the ranking of the alternative CRCs changes slightly.

4.3. *Discussion*

This research conducts two reverse logistics applications by using a hybrid fuzzy approach based on modified VIKOR method and modified GRA method in fuzzy environments. The results of the analysis show the feasibility and validity of the proposed approach. In addition, the proposed approach has the advantages in the applications as follows:

(1) It is worth pointing out that for simple applications, the new relative closeness coefficient can be reduced to a single relative closeness coefficient on the basis of distance measure or similarity measure by selecting the values of parameter α.
(2) For the hierarchical evaluation indexes in some practical problems, Analytic Hierarchy Process (AHP)[47] or Analytic Network Process (ANP)[48] can be easily incorporated into our proposed method to obtain the values or weights of the low-level attributes for the goal of the problems, and then the proposed approach can be applied to get the final evaluation result.
(3) The proposed approach can be easily combined with ANP,[48] principal component analysis (PCA),[49] choquet integral,[50,51] etc., to deal with the case of the correlations among the attributes.

5. Conclusion and Remarks

Multi-attribute decision-making methods can be used to solve the problems with uncertain and imprecise data in group decision-making as FMAGDM. In this paper, we propose a hybrid fuzzy approach to solve FMAGDM problems and establish a new relative closeness coefficient by combining the distance measure and the similarity measure to rank the set of feasible alternatives.

The proposed approach can effectively grasp the ambiguity and vagueness of the information available by using fuzzy numbers to evaluate the fuzzy ratings of alternatives and fuzzy weights of attributes. It has also produced satisfactory results as observed in the applications of the method to reverse logistics system design. This method avoids the defuzzification of fuzzy numbers, and thus causes no loss of information. The distance of two fuzzy numbers calculated by the vertex method is adopted in the modified VIKOR method and the modified GRA method to reduce computations for fuzzy numbers operations. Moreover, the results of the proposed method are crisp numbers, which can avoid inconsistent results by using a fuzzy ranking method to rank the alternatives' final evaluation values. Finally, it should be pointed out that the DMs have responsibility to select the value of v appropriately according to their preferences, ways of making decisions, and the features of the specific problems.

Although the proposed approach is only applied to the reverse logistics environment in this paper, it can also be applied to solve other problems such as information system project selection and supplier selection problems in different areas of management decision-making.

FMAGDM problems can be better solved if a decision support system in a fuzzy environment can be developed. This will be the topic for our future research. Furthermore, because the proposed approach only focuses on single period evaluations, its extension to solving dynamic/multiperiod multi-attribute decision-making problems[52–54] will be another topic for future studies in this area.

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